Are We Speaking the Same Language?

A. Introduction

1. Why this interests me.
2. Where do definitions come from?
3. Math Dictionaries related to this discussion

A. Introduction

2. Where do definitions come from?

Definition:

An agreement to use something (a symbol or set of words) as a substitute for something else, usually for some expression that is too lengthy to write easily or conveniently. **MD5**

A. Introduction

Line: (Barron's)

A line is a straight set of points that extends off to infinity in two directions.

The term "line" is one of the basic undefined terms of Euclidean geometry, so it is not possible to give a rigorous definition of a line.

You will have to use your *intuition* as to what it means for a line to be *straight*.

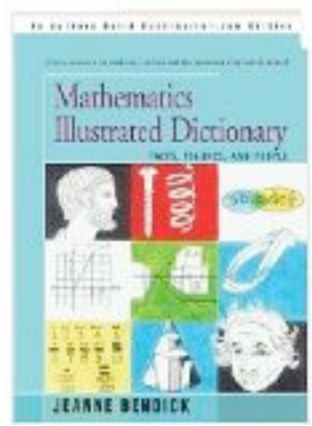
A. Introduction

3. Math Dictionaries related to this discussion

Mathematics Dictionary, 5th Ed.
James and James

MD5

Mathematics Illustrated Dictionary
Jeanne Bendick



Illustrated

Are We Speaking the Same Language?

A. Introduction

3. Math Dictionaries related to this discussion

Dictionary of Mathematics Terms
Douglas Downing

Barron's

Facts on File: Algebra Handbook
Deborah Todd

F on F

A. Introduction

Facts on File has this circular definition:

A **term** is any number, variable, or group of numbers and variables that form a monomial.

A **monomial** is any expression that consists of just one term. Expressions with more than one term are types of polynomials.

A. Introduction

Some multiple meaning words: **Base**, **degree**, **term**.

A. Introduction

Term: (MD5)

For an expression which is written as the sum of several quantities, each of these quantities is called a **term** of the expression;

e.g., in $x^2 + y\sin x - \frac{x+1}{y-1} - (x+y)$,

the terms are x^2, $y\sin x$, $-\frac{x+1}{y-1}$, and $-(x+y)$.

A. Introduction

Term:

Terms of a fraction	Constant term
Terms of a proportion	Transcendental term
A term of an equation	Terms of a sequence
Terms of a polynomial	Terms of endearment
Algebraic term	Term life insurance.

A. Introduction

Base:

Base of a triangle (one dimensional base)

Base of a cylinder (two dimensional base)

Base of an exponential expression

Base of a logarithm

Base in a proportion/percent (the percentage is a percent of the base)

Base in a numbering system (such as base-10 and base-2)

Are We Speaking the Same Language?

A. Introduction

Degree:

Degree of an angle

Degree of a monomial

Degree of a polynomial

Degree of an equation

Degree in temperature

Degree of freedom

Is it any wonder that students sometimes get confused by the language of math?

A. Introduction

The word **difference** is confusing to me!

If Farmer Agrico has 15 sheep and Farmer Bauer has 23 sheep, then the *difference* between the numbers of sheep is 8, no matter how you look at it.

I.e., the mathematical notion of *difference,* which could be negative, doesn't fully align with the common sense usage of the word.

It is necessary in calculating the slope, but the slope is defined as the ratio of the ***change*** in y to the ***change*** in x.

B. Correct the Definitions

1. How important is it to be accurate?
2. Get in groups of 3 or 4
3. Determine whether the given definition (handout) is accurate. If it is not, how should it be corrected?

Are We Speaking the Same Language?

B. Correct the Definitions

Measures of central tendency

A measure of central tendency indicates a middle or typical value of a group of numbers.

Examples of measures of central tendency are the *mean* (or *average*), the *median*, and the *mode*. (Mode/Median are not averages.)

B. Correct the Definitions

Improper rational expression

An improper rational expression is one in which the degree of the numerator is <u>greater than or equal to</u> the degree of the denominator.

Improper rational expressions:

$$\frac{x^2-6x+8}{x-1} \qquad \frac{3x+5}{6x-1}$$

Proper rational expression:

$$\frac{3x-4}{3x^2+2x-8}$$

Are We Speaking the Same Language?

C. Some Distinctions Between Dictionaries

Monomial:

F on F

Any expression that consists of just one term. (Expression with more than one term are types of polynomials.)

Illustrated

A monomial can be an integer, or a variable. It can be the product of an integer and variables.

Are We Speaking the Same Language?

C. Some Distinctions Between Dictionaries

Monomial:

Barron's

An algebraic expression that does not involve any additions or subtractions.

MD5

An algebraic expression consisting of a single term which is a product of numbers and variables.

C. Some Distinctions Between Dictionaries

Polynomial:

F on F

Any expression that consists of a string of monomials.

Illustrated

A monomial or the algebraic sum of monomials.

Barron's

A polynomial in x is an algebraic expression of the form $a_n x^n + a_{n-1} x^{n-1} + \cdot\cdot\cdot + a_1 x + a_0$. where a_i are constants that are the coefficients of the polynomial.

Can a monomial (including a constant) be a polynomial?

In a polynomial, is the constant term, a_0, a coefficient?

Are We Speaking the Same Language?

C. Some Distinctions Between Dictionaries

Polynomial: MD5

A polynomial in **one variable** (usually called a simple polynomial) of degree n, is a rational integral algebraic expression of the form

$$a_0x^n + a_1x^{n-1} + \cdot \cdot \cdot + a_{n-1}x + a_n,$$

where a_i is a complex number (real or imaginary), and n is a non-negative integer.

Constants, then, are polynomials of degree 0, except that the constant 0 is not assigned a degree.

A polynomial in **several variables** is an expression which is the sum of terms, each of which is the product of a constant and various non-negative powers of the variable.

Are We Speaking the Same Language?

C. Some Distinctions Between Dictionaries

Degree of a Polynomial:

F on F and **Barron's**

Degree of highest exponent in a polynomial.

Illustrated

The degree of the monomial term of highest degree.
(**Degree of a monomial** is the sum of the exponents of the variables.)

MD5

The degree of its highest-degreed term.
(**Degree of a term:** A term in several variables has degree equal to the sum of the exponents of its variables.)

Are We Speaking the Same Language?

C. Some Distinctions Between Dictionaries

Like Terms:

F on F

Like terms are any terms that have the same variable but different coefficients.

Illustrated

Like terms are terms that are the same with respect to the variable(s) and exponent(s) of these variable(s).

Are We Speaking the Same Language?

C. Some Distinctions Between Dictionaries

Like Terms:

Barron's

Two terms are **like terms** if all parts of both terms except for the numerical coefficients are the same.

MD5

Like terms are terms that contain the same variables, each variable of the same kind being raised to the same power.

Are We Speaking the Same Language?

C. My Definitions Related to Polynomials

1. A polynomial **term** is a constant or is the product of a constant and one or more variable factors.

   The numerical factor of a term is called the term's **coefficient**, and the product of variable factors is referred to as the term's **variable structure**.

2. A term's **variable structure** is either one variable, with its own whole number exponent, or the product of two or more variables, each with its own exponent. A constant term has no variable factors; we can say that in a constant term, all variables have an exponent of 0.

Are We Speaking the Same Language?

C. My Definitions Related to Polynomials

3. Two or more terms are considered to be **like terms** if their variable structures are *exactly* the same.

4. The **degree of a term** is the number of variable factors in the term. In general, the degree of a term is the sum of all of the exponents in the term's variable structure. Every non-zero constant term has a degree of 0 because it has no variable factors.

5. A **polynomial** is either a single polynomial term or is the sum of two or more such terms.

Are We Speaking the Same Language?

C. My Definitions Related to Polynomials

6. A polynomial is in **descending order** when the terms are written, according to their *degree*, from highest to lowest.

   If a polynomial has two or more unlike terms with the same degree, it is typical to create descending order using the powers of the variable that is alphabetically first.

7. The first term of a polynomial *in descending order* is called the **leading term** (or **lead term**) of the polynomial; the coefficient of the leading term is called the **leading** (or **lead**) **coefficient**.

8. The **degree of a polynomial** is *the same as the degree of its leading term.*

C. Some Distinctions Between Dictionaries

Simplify:

F on F

Simplifying Removing grouping symbols and combining like terms to bring the equation or sentence to its simplest form.

Illustrated

Simplify To write a shorter form of a numeral or algebraic expression.

Barron's

(none given)

Are We Speaking the Same Language?

C. Some Distinctions Between Dictionaries

Simplify: MD5

One of the most indefinite terms used seriously in mathematics. It's meaning depends upon the operation as well as the expression at hand and its setting.

The **simplified** form of an expression, quantity, or equation can mean either (1) the briefest, least complex form, or (2) the form best adapted to the next step to be taken in the process of seeking a certain result.

E.g., if one desired to factor $x^4 + 2x^2 + 1 - x^2$, to collect the x^2 terms would be foolish, since it would conceal the factors.

Are We Speaking the Same Language?

C. Some Distinctions Between Dictionaries

Adjacent Angles:

F on F

Either of two angles that share a common side and vertex.

Barron's

Two angles are adjacent if they share the same vertex and have one side in common between them.

MD5

Two angles having a common side and vertex and lying on opposite sides of their common side.

Illustrated

Two angles in the same plane that have a common side and vertex. *They have no interior points in common.*

Are We Speaking the Same Language?

D. Some questions related to math definitions

1. Is $3x^2 + 3x - 3$ a prime polynomial?
2. Is $x + 2 - \sqrt{2}$ a polynomial?
3. What does ***-gon*** mean as a root word?
4. What does it mean to **cancel**?
5. What does -1 mean?
6. Why is 1 not a prime number?
7. Are **power** and exponent synonymous?
8. Is $x^2 + y^2 = 5$ a **quadratic equation?**

Are We Speaking the Same Language?

D. Is $3x^2 + 3x - 3$ a prime polynomial? **MD5**

An **irreducible radical** is one that cannot be written in an equivalent rational form.

E.g. $\sqrt{6}$, $\sqrt{x}$, $\sqrt[3]{4}$ and are all *irreducible.*

but $\sqrt{4}$ and $\sqrt[3]{x^3}$ are reducible.

Are We Speaking the Same Language?

D. Is $3x^2 + 3x - 3$ a prime polynomial? MD5

An **irreducible polynomial** is a polynomial that cannot be written as the product of two polynomials with degrees at least 1 and having coefficients in some given domain or field. Unless otherwise stated, *irreducible* means irreducible in the field of the coefficients of the polynomial.

E.g., the binomial $x^2 + 1$ is irreducible in the field of real numbers, although in the field of complex numbers, it can be factored as $(x + i)(x - i)$.

In elementary algebra, it is understood that an irreducible polynomial is a polynomial that cannot be factored into factors having rational coefficients.

Are We Speaking the Same Language?

D. Is $3x^2 + 3x - 3$ a prime polynomial? **MD5**

A **prime polynomial** is a polynomial which has no polynomial factors except itself and constants.

E.g., $3x^2 + 3x - 3$ is a prime polynomial:

$$3x^2 + 3x - 3 = 3(x^2 + x - 1)$$

Are We Speaking the Same Language?

D. Is $x + 2 - \sqrt{2}$ a polynomial? **MD5**

If it is, then $x^2 + 4x + 2$ is factorable:

$$x^2 + 4x + 2 = (x + 2 - \sqrt{2})(x + 2 + \sqrt{2})$$

This works fine if the field of the coefficients is real numbers.

Are We Speaking the Same Language?

D. What does *-gon* mean as a root word? **MD5**

Its etymology is from the Greek word *gony*, which means *knee*, and the root means *angle*.

So, a ***polygon*** is a <u>many-angled</u> closed figure, and

isogonal means *having <u>equal angle</u> measures.*

Are We Speaking the Same Language?

D. What does it mean to **cancel**?

MD5

(1) To **cancel** is to divide factors out of the numerator and denominator of a fraction. (2) two quantities of opposite sign but numerically equal are said to **cancel** when added.

Illustrated

To **cancel** is to add equal quantities to both members of an equation, or to divide out a factor common to both term of a fraction.

F on F

To **cancel** is to divide the numerator and denominator of a fraction by a common factor.

Barron's

(none)

Are We Speaking the Same Language?

D. What does it mean to **cancel**?

I say:

Canceling is the process of applying an inverse operation or function.

Canceling is helpful when describing the following simplifications:

$$\left(\sqrt{x+3}\right)^2 = x+3$$

$$\log_b\left(b^n\right) = n$$

$$\frac{d\int f(x)dx}{dx} = f(x)$$

Are We Speaking the Same Language?

D. What does -1 mean?

Besides being a number on the number line, -1 means "inverse."

We see this in the following ways:

1. $-1 \cdot a = -a$, the opposite of a, the additive inverse.

2. $a^{-1} = \frac{1}{a}$, the reciprocal of a, the multiplicative inverse.

3. $f^{-1}(x)$ represents the inverse of a function.

Each use of -1, as an inverse, must be read in context. For example, $a^{-1} \neq -a$ and $f^{-1}(x)$ does not mean the reciprocal of $f(x)$.

Are We Speaking the Same Language?

D. Why is 1 not a prime number?

Illustrated

A **prime number** is a natural number that has no other factors except 1 and itself. 2, 3, 5, 7, 11, 13, 17, 19, 23 ... are prime numbers. (1 is usually not included in the set of prime numbers.)

Barron's

A **prime number** is a natural number that has no integer factors other than itself and 1. The smallest prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41.

Are We Speaking the Same Language?

D. Why is 1 not a prime number?

F on F

Any number that is divisible only by 1 and itself is call a **prime number**. For example, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, and so on.

MD5

A **prime** is any integer, *p*, that is not 0 or ±1 and is divisible by no integers except ±1 and ±*p*. Sometimes a prime is required to be positive.

My definition

A natural number is **prime**, or is a **prime number**, if it has exactly two distinct, whole number factors, 1 and itself.

Are We Speaking the Same Language?

D. So, why is 1 not a prime number?

I say:

1 is not a prime number because, if it was, then the prime factorization of a whole number would not be unique.

For example,

$12 = 3^1 \cdot 2^2 \cdot 1^3$

$12 = 3^1 \cdot 2^2 \cdot 1^4$

$12 = 3^1 \cdot 2^2 \cdot 1^5$

and so on.

Are We Speaking the Same Language?

D. Are **power** and exponent synonymous?

Barron's

A **power** of a number indicates repeated multiplication. For example, "*b* to the third power" means "*b* multiplied by itself three times" (*b* x *b* x *b*). Powers are written with little raised numbers known as *exponents*.

Illustrated

2^3 is called a **power**. It is the third power of 2 and it is equal to 8. in general, b^n is a number and is called the nth power of b.

Are We Speaking the Same Language?

D. Are **power** and exponent synonymous?

MD5

An exponent is a number placed at the right of and above a symbol. The value assigned to the symbol with this exponent is called a **power** of the symbol; although, *power* is sometimes use in the same sense as *exponent*.

My definition

A **power** is the result of applying an exponent to its base. For example, $2^3 = 8$ means "the third power of 2 is 8."

A **power** is also the exponential expression. For example, the fourth power of 10 can be written as 10,000 or as 10^4.

Are We Speaking the Same Language?

D. Is $x^2 + y^2 = 5$ a **quadratic equation?**

Illustrated

A **quadratic equation** is an equation of second degree. Equations of the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers, $a \neq 0$, are called *quadratic equations*.

F on F

A **quadratic equation** is any equation with only a squared term as its highest term.

MD5

A **quadratic equation** is a polynomial equation of the second degree.

Barron's

A **quadratic equation** is an equation involving the second power, but no higher power, of an unknown.

Are We Speaking the Same Language?

E. Some unusual mathematical terms

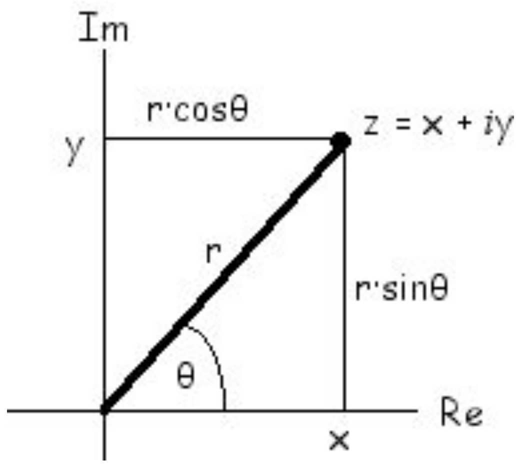
Do You Know What This Is? These all come from MD5

Aliquot part

Any exact divisor (factor) of a quantity. E.g., 2 and 3 are aliquot parts of 6.

Argand diagram

Two perpendicular axes, on one of which real numbers are represented, and on the other pure imaginaries, thus providing a reference for graphing complex numbers.



Are We Speaking the Same Language?

E. Some unusual mathematical terms

Do You Know What This Is? These all come from MD5

Perfect, defective (or deficient), and abundant numbers

Perfect number: A number equal to the sum of all divisors, except itself. (Examples: 6 and 28)

Defective number: The sum of all divisors (other than itself) is less than the number. (Examples: 10 and 32)

Abundant number: The sum of all divisors (other than itself) is greater than the number . (Examples: 24 and 60)

E. Some unusual mathematical terms

Do You Know What This Is? These all come from MD5

Amicable numbers

Two numbers, each of which is equal to the sum of all the exact divisors of the other, except the number itself.

For example 220 and 284.

The divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, and 110, the sum of which is 284.

The divisors of 284 are 1, 2, 4, 71, and 142, the sum of which is 220.

There are 236 known amicable pairs for which the smaller of the two is less than 10^8.

Are We Speaking the Same Language?

E. Some unusual mathematical terms

Do You Know What This Is? These all come from MD5

Explementary angles

Two angles whose sum is 360°.

Conjugate angles

Two angles whose sum is 360°. Such angles are sometimes said to be *explements* of each other.

Reflex angle

An angle greater than 180° but less than 360°.

Perigon

An angle of 360°. (Also called a *round angle*.)

Isogonal

Having equal angles.

E. Some unusual mathematical terms

Do You Know What This Is? These all come from MD5

Flexion

A name sometimes used for the rate of change of the slope of a curve; the second derivative of a function. (**Flex** means *inflection*.)

Continued equality

three or more quantities set equal by means of two or more equality signs in a continuous expression.

Surd

A sum with one or more irrational indicated roots as addends.

For example, $\sqrt{3} + 2\sqrt{5}$ and $1 - \sqrt{7} + \sqrt[3]{9}$

E. Some unusual mathematical terms

Do You Know What This Is? These all come from MD5

Solidus

A slant line used to indicate division in a fraction, such as 3/4 or a/b. Also used in dates, such as 3/6/10.

Triangular numbers

The numbers 1, 3, 6, 10, 15, ... those that can form a triangle by that number of dots.

Vigesimal

Having to do with 20, such as a vigesimal numbering system (used by the Aztecs).

Are We Speaking the Same Language?

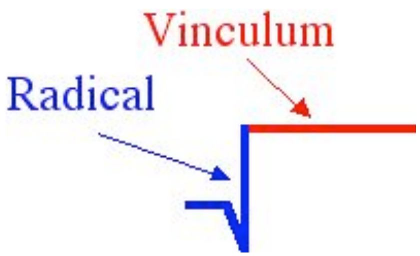
E. Some unusual mathematical terms

Do You Know What This Is? From MD5 and others

Vinculum

A bar used to indicate an aggregation; a grouping symbol.

It is typically used as an "over bar," such as that used with a radical.



It is also used as to separate the numerator from the denominator in a fraction.

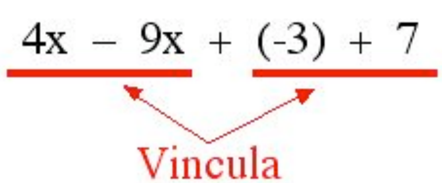
Are We Speaking the Same Language?

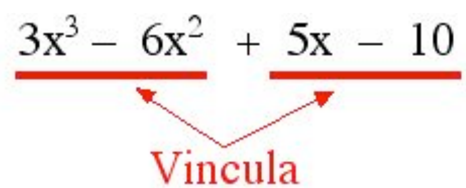
E. Some unusual mathematical terms

Do You Know What This Is? From MD5 and others

Vinculum

I'd like to all see it as an "under bar, as in grouping terms in an algebraic expression. (Note: <u>A plus sign is required between groupings</u>.)





Are We Speaking the Same Language?

F. Can there be new definitions?

I propose the following new definitions:

Quadrinomial: a four-term polynomial.

Axial points: Points on an axis. In the x-y-plane, an **axial point** is any point that has at least one 0 coordinate.

Betweenness inequality: $a < x < b$.
This means that x is *between* a and b. For example, $|2x - 7| < 5$ creates a betweenness inequality: $-5 < 2x - 7 < 5$

Parent function: A function in which the argument is just x (the independent variable). For example, these are parent functions:

$$y = x^2 \qquad y = \sqrt{x} \qquad y = \frac{1}{x}$$

Are We Speaking the Same Language?

F. Can there be new definitions?

I propose the following new definitions:

Double negative: Two negative signs, or a minus sign and a negative sign, without any term or operation between them.

Support for this definition comes from **MD5**:

> **Law of signs:** In addition and subtraction, two adjacent like signs can be replaced by a positive sign, and two adjacent unlike signs can be replaced by a negative sign.

Are We Speaking the Same Language?

F. Can there be new definitions?

I propose the following new definitions:

Variable structure: In a polynomial term, the product of all of its variable factors.

This allows us to talk easily about

a) **like terms:** two terms with the same variable structure

b) **the degree of a term:** the number of variable factors in the term's variable structure

Are We Speaking the Same Language?

F. Can there be new definitions?

I propose the following new definitions:

Main operation: the last operation to be applied in an expression, according to the order of operations.

Some benefits of the main operation are:

1. When translating from English to Algebra (or vice-versa), the main operation is the one written first. For example,

a) The sum of 5 and the product of 2 and a number is $5 + 2x$.

b) The product of 5 and the sum of 2 and a number is $5(2 + x)$.

F. Can there be new definitions?

I propose the following new definitions:

Main operation: the last operation to be applied in an expression, according to the order of operations.

Some benefits of the main operation are:

2. When solving an equation involving at least two operations, it is the main operation that should be cleared first:

a) In $3x - 15 = 21$, the main operation is subtraction, so to isolate the variable we should clear the constant first by adding its opposite to each side.

b) In $\sqrt{4x - 11} = 3$, the main operation is the square root, so we should clear the radical first by squaring each side.

Are We Speaking the Same Language?

F. Can there be new definitions?

I propose the following new definitions:

Main operation: the last operation to be applied in an expression, according to the order of operations.

3. Distribution changes the main operation.

a) $6(x - 4) = 6x - 24$, the main operation changes from multiplication to subtraction.

b) $\left(x^5 \cdot y^3\right)^2 = x^{10} \cdot y^6$, the main operation changes from an exponent, 2, to multiplication.

Are We Speaking the Same Language?

F. Can there be new definitions?

I use this number line definition of *number* in my writing:

> Every non-zero number has both a numerical value and a direction.

To justify this definition, I refer to **MD5**:

Numerical value: the same as the absolute value

Negative direction is the direction opposite the direction that has been chosen as positive.

F. Can there be new definitions?

I use this number line definition of *number* in my writing:

> Every non-zero number has both a numerical value and a direction.

To justify this definition, I refer to **MD5**:

Directed numbers: Numbers having signs, positive or negative, indicating that the negative numbers are to be measured, geometrically, in the direction opposite to that in which the positive are measured when the numbers are considered to be points on the number line. *Syn.*, signed numbers, algebraic numbers.

Are We Speaking the Same Language?

F. Can there be new definitions?

I want to use this definition in my writing:

Variable term:

In an equation, any term that contains the variable to be solved for is called a ***variable term***.

To this end, if we are to solve for W in M = h + kW, then I would like the variable term to be kW.

For example, in the literal equation, M = h + kW, if we are to solve for W, then the *variable term* is kW.

Are We Speaking the Same Language?

F. Can there be new definitions?

Get into groups of 3 or 4 and discuss

1. Words that you'd like to see used and defined.

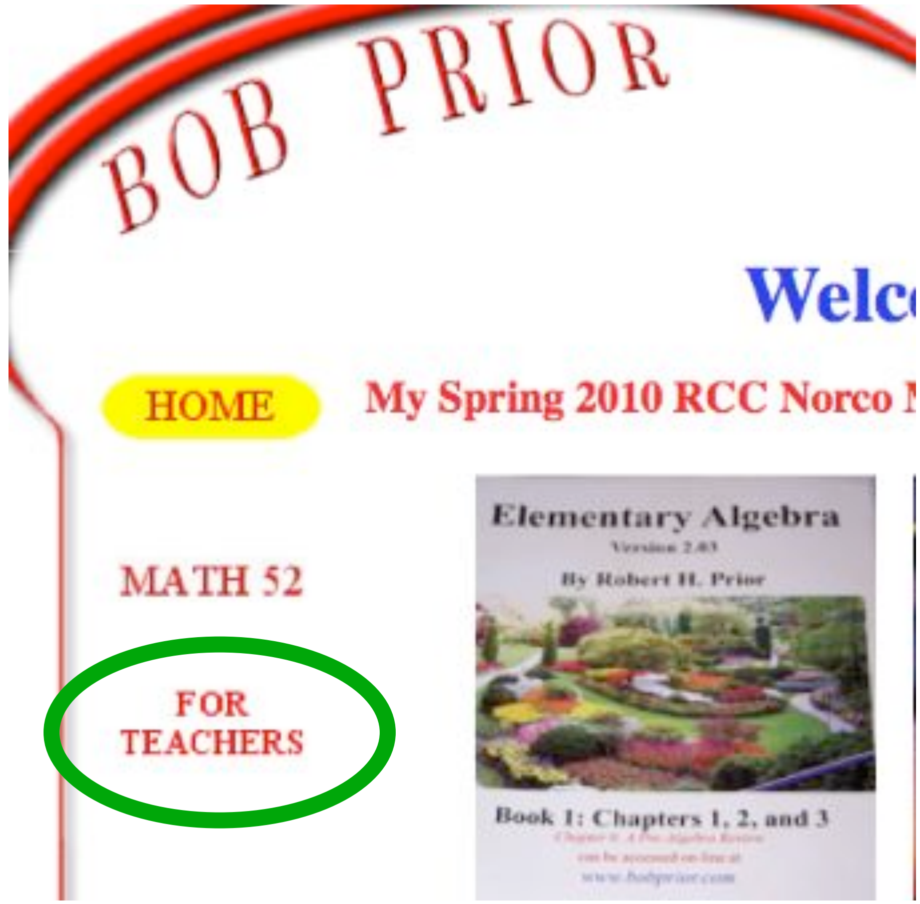
2. Concepts you'd like to have a mathematical word for.

G. On-Line Resources

My Website:

http://bobprior.com

Click on the link that says "For Teachers."



Are We Speaking the Same Language?

G. On-Line Resources http://bobprior.com/forteachers

1. Are We Speaking the Same Language? (Presented at CMC3-South)

This discussion is about mathematics dictionaries and their similarities and differences. The most surprising is the differences. It turns out that not all math terms are defined the same. Also, is it too late to invent new terms?

This is a pdf of my PowerPoint presentation at the CMC3-South Spring 2010 Conference.

Are We Speaking the Same Language?

Here is a link to a web page that has, itself, links to many on-line math dictionaries and glossaries.

A list of On-line Mathematical Dictionaries

Here is a link to a web page that shows the earliest known uses of many mathematical terms.

Earliest Known Uses of Mathematical Terms