# Dividing Whole Numbers

## INTRODUCTION

Consider the following situations:

1. Edgar brought home 72 donut holes for the 9 children at his daughter's slumber party. How many will each child receive if the donut holes are divided equally among all nine?

2. Three friends, Gloria, Jan, and Mie-Yun won a small lottery prize of $1,450. If they split it equally among all three, how much will each get?

3. Shawntee just purchased a used car. She has agreed to pay a total of $6,200 over a 36-month period. What will her monthly payments be?

4. 195 people are planning to attend an awards banquet. Ruben is in charge of the table rental. If each table seats 8, how many tables are needed so that everyone has a seat?

Each of these problems can be answered using division. As you'll recall from Section 1.2, the result of division is called the quotient. Here are the parts of a division problem:

**Standard form:** dividend ÷ divisor = quotient

**Long division form:**

$$\begin{array}{r} \text{quotient} \\ \text{divisor}\,\overline{)\,\text{dividend}} \end{array}$$

**Fraction form:** $\dfrac{\text{dividend}}{\text{divisor}} = \text{quotient}$

We use these words—dividend, divisor, and quotient—throughout this section, so it's important that you become familiar with them. You'll often see this little diagram as a continual reminder of their proper placement.

$$\begin{array}{r} \text{quotient} \\ \text{divisor}\,\overline{)\,\text{dividend}} \end{array}$$

Here is how a division problem is read:

In the standard form: $15 \div 5$ is read "15 divided *by* 5."

In the long division form: $5\,\overline{)\,15}$ is read "5 divided *into* 15."

In the fraction form: $\dfrac{15}{5}$ is read "15 divided *by* 5."

$$\begin{array}{r|l} & \text{quotient} \\ \hline \text{divisor} & \text{dividend} \end{array}$$

**Example 1:** In this division problem, identify the dividend, divisor, and quotient.

$$36 \div 4 = 9, \qquad \begin{array}{r} 9 \\ 4\overline{)36} \end{array} \qquad \text{or} \qquad \frac{36}{4} = 9$$

**Answer:** For each, the dividend is __36__ , the divisor is _4_ , and the quotient is _9_ .

**YTI #1** In each division problem, identify the dividend, divisor, and quotient. Use Example 1 as a guide.

a) $28 \div 7 = 4$ dividend: __________ divisor: __________ quotient: __________

b) $\frac{40}{8} = 5$ dividend: __________ divisor: __________ quotient: __________

c) $\begin{array}{r} 6 \\ 2\overline{)12} \end{array}$ dividend: __________ divisor: __________ quotient: __________

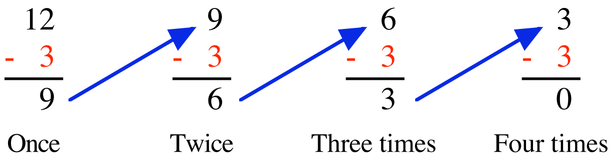
## WHAT IS DIVISION?

**~ Begin Instructor Insights ~**

This demonstration of division as repeated subtraction is only for the students' understanding. It is not intended that students will duplicate this process in the exercises.

**~ End Instructor Insights ~**

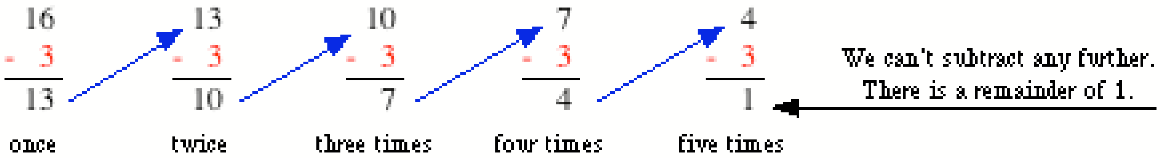
**Division** is *the inverse operation of multiplication*. Just as multiplication is repeated addition, division can be thought of as repeated subtraction. For example, to find out how many times 3 will divide into 12, we can subtract 3 repeatedly until we get to 0:



We see that 3 will divide into 12 exactly four times. In other words, $12 \div 3 = 4$ (exactly). We often use the phrase "divides evenly into" when speaking of *exact* division. In the example $12 \div 3$, we can say that "3 divides evenly into 12 four times."

Another term that means *divides evenly* is "divisible." We can say, for example, that 12 is divisible by 3.

Sometimes this repeated subtraction process will not result in 0, and we'll have a little left over, a *remainder*. For example, 3 will divide into 16 five times with 1 left over:

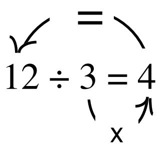


We can say that 3 does *not* divide into 16 exactly.

The remainder must be smaller than the divisor. For example, in repeatedly subtracting 3 from 16, we can't stop when we get to 13, 10, 7, or 4 because it is still possible to subtract 3 at least one more time.

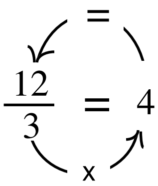
For exact division, we can use a circular argument of inverses to see how multiplication and division work together:

**Standard form**



3 x 4 = 12

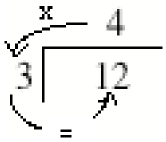
**Fraction form**



3 x 4 = 12

and

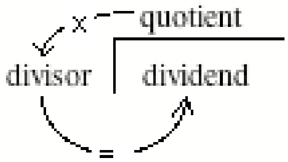
**Long division form**



4 x 3 = 12

This circular argument says:

**quotient x divisor = dividend**



Because of the circular nature of division and multiplication, we can use multiplication to verify the accuracy of our division result, the quotient.

## SHORT DIVISION

When we are able to divide in one step, as demonstrated in Examples 2 and 3, we call it **short division**. When the division is not obvious (it cannot be done in just one step), we call it **long division**.

In demonstrating short division, we can use the standard form (as shown in Example 2) or the long division form (as shown in Example 3).

**Example 2:** Use short division to evaluate. Verify each answer by multiplying the divisor and the quotient.

a) $\frac{35}{7} = \_\_\_\_$ b) $5 \div 5 = \_\_\_\_$ c) $0 \div 4 = \_\_\_\_$

**Procedure:** Think about what number will multiply by the divisor to get the quotient. In other words, use the circular argument for division, divisor x quotient = dividend, to find the quotient.

**Answer:**

a) $\frac{35}{7} = \underline{\ 5\ }$ because $7 \times \underline{\ 5\ } = 35$

b) $5 \div 5 = \underline{\ 1\ }$ because $5 \times \underline{\ 1\ } = 5$ Rule: a natural number divided by itself is 1.

c) $0 \div 4 = \underline{\ 0\ }$ because $4 \times \underline{\ 0\ } = 0$ Rule: 0 divided by a natural number is 0.

Example 2 demonstrates two basic principles of division:

1) A natural number divided by itself is 1.
2) 0 divided by a natural number is 0.

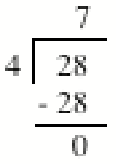
**YTI #2** Use short division to evaluate. Verify each answer by multiplying the divisor by the quotient, as in Example 2.

a) $50 \div 5 = \_\_\_\_\_\_$ b) $\frac{18}{3} = \_\_\_\_\_\_$ c) $4 \div 4 = \_\_\_\_\_\_$

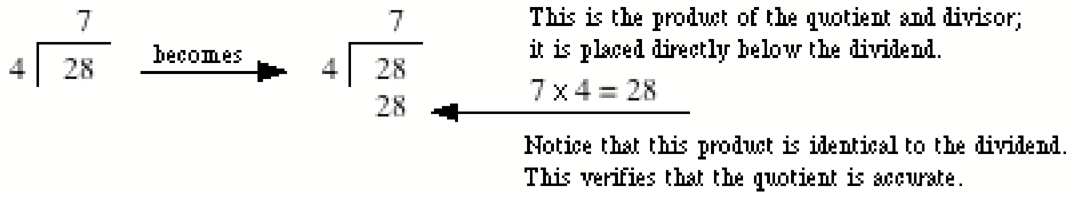
d) $\frac{40}{5} = \_\_\_\_\_\_$ e) $0 \div 2 = \_\_\_\_\_\_$ f) $7 \div 1 = \_\_\_\_\_\_$

In standard form, to verify that $28 \div 4 = 7$, we might write $\underline{\text{ because } 4 \times 7 = 28.}$

To verify the same division using the long division form, we can show the product of 4 and 7 *below* the dividend:



And, because they are the same, the result is 0 when we subtract:



This is the product of the quotient and divisor; it is placed directly below the dividend.

Notice that this product is identical to the dividend. This verifies that the quotient is accurate.

**Example 3:** Use short division to evaluate. Verify each answer by multiplying the divisor and the quotient, and write that product below the dividend and subtract.

a) $2\overline{)18}$ b) $10\overline{)60}$ c) $1\overline{)3}$

**Procedure:** To find the quotient, think about what number will multiply the divisor to get the dividend. In other words, use the circular argument for division,

divisor x quotient = dividend, to find the quotient.

**Answer:**

a) $\begin{array}{r} \boxed{9} \\ 2\overline{)18} \\ -18 \\ \hline 0 \end{array}$ ← This 0 indicates that 2 divides evenly into 18.

b) $\begin{array}{r} \boxed{6} \\ 10\overline{)60} \\ -60 \\ \hline 0 \end{array}$

c) $\begin{array}{r} \boxed{3} \\ 1\overline{)3} \\ -3 \\ \hline 0 \end{array}$

Rule: any number divided by 1 is itself.

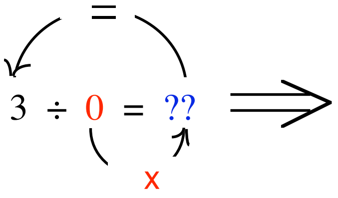
**YTI #3** Use short division to evaluate. Verify each answer by multiplying the divisor by the quotient, as in Example 3.

a) $7\overline{)49}$ b) $6\overline{)54}$ c) $8\overline{)8}$ d) $1\overline{)6}$

$\begin{array}{r|l} & \text{quotient} \\ \hline \text{divisor} & \text{dividend} \end{array}$

## CAN THE DIVISOR EVER BE 0?

We can *never* divide by 0 (zero). We have seen that 0 can be the dividend, but it can never be the divisor. The circular argument shows us why:



$0 \times ?? = 3$

There is no number that fits here because 0 x any number = **0**, never 3.

In general, we say that **$a \div 0$, or $\frac{a}{0}$, is undefined**.

As an example, you might think of division by 0 in terms of distributing equal lottery winnings among a “group” of people:

| Lottery winnings | ÷ | # of people | = | Each person gets | |
|---|---|---|---|---|---|
| $0 | ÷ | 3 | = | $0 | $0 \div 3 = 0$ |
| $35 | ÷ | 0 | = | ?? | $35 \div 0 = ??$<br>(impossible) |

You can’t distribute $35 among 0 people. It isn’t possible.

### THE REMAINDER

Earlier in this section we saw that 3 does not divide evenly into 16; that when we subtracted 3 repeatedly, we ended with a remainder of 1.

We also saw that 3 does divide evenly into 12 four times; when we subtracted 3 repeatedly, we were able to get an end result of 0.

If the divisor divides evenly into the dividend, then the remainder is 0 (there is no remainder).

Here is a simple example to illustrate how the remainder is found in the division process.

Consider $17 \div 5$:

5 does not divide into 17 evenly, so we will have a remainder. Using the long division form, $5\overline{)17}$, we can start the quotient by choosing a number that, when multiplied to 5, will give a product close to 17.

The number we choose to start the quotient can’t be too large—it can’t give us a product more than the dividend (because then we can’t subtract to find the remainder); nor can it be too small—the remainder we get must be less than the divisor. Let’s try 4:

$$\begin{array}{r} 4 \\ 5\overline{)17} \\ -\,20 \\ \hline ?? \end{array}$$

It turns out that 4 is too large, because 4 x 5 = 20 and can’t be subtracted from 17.

If we try to start the division process with 2, we’ll get a remainder that is larger than the divisor, and that is not allowed.

$$\begin{array}{r} 2 \\ 5\overline{)17} \\ -\,10 \\ \hline 7 \end{array}$$

This remainder, 7, is greater than 5. This means that the quotient, 2, was too small.

Instead, if we try 3, it gives us a product that is a little less than 17, and a remainder that is smaller than the divisor:

$$\begin{array}{r} 3 \\ 5\overline{)17} \\ -\ 15 \\ \hline 2 \end{array}$$

This remainder is just right.

$$\begin{array}{r} 3\ \text{r}\ 2 \\ 5\overline{)17} \\ -\ 15 \\ \hline 2 \end{array}$$

The remainder is shown to the right of the quotient

Notice that the remainder is shown next to the quotient—a whole number—and is abbreviated by the letter **r**. When a remainder exists, we will see the long division answer as ...

$$\begin{array}{l} \qquad\qquad \text{quotient r remainder} \\ \text{divisor}\overline{)\text{dividend}\qquad\qquad} \end{array}$$

**Example 4:** Divide. If—after multiplying the quotient and the divisor—there is a remainder (other than 0), show it next to the quotient.

a) $6\overline{)27}$ b) $7\overline{)42}$

**Procedure:** If the divisor doesn't divide evenly into the dividend, then think about what number it will divide into evenly.

**Answer:**

a) $$\begin{array}{r} 4\ \text{r}\ 3 \\ 6\overline{)27} \\ -\ 24 \\ \hline 3 \end{array}$$

6 x **4** = 24 is less than 27 and 6 x **5** = 30 is too much.

The quotient is 4 and the remainder is 3.

(This remainder is less than the divisor, 6, so we have divided properly.)

b) $$\begin{array}{r} 6 \\ 7\overline{)42} \\ -\ 42 \\ \hline 0 \end{array}$$

7 x **6** = 42, so 42 is divisible by 7, and the remainder is 0.

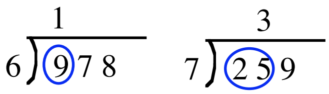
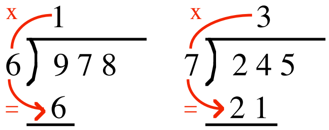
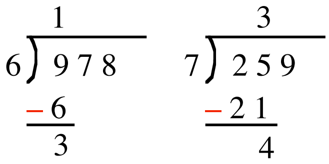
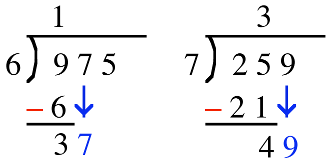
**YTI #4** Divide. Be sure to show any remainder (other than 0) next to the quotient. (Do these in pencil. Have an eraser handy in case your first try gives a quotient that is either too large or too small.) Use Example 4 as a guide.

a) $7\overline{)60}$ b) $8\overline{)15}$ c) $4\overline{)28}$ d) $11\overline{)70}$

## THE LONG DIVISION ALGORITHM

As you might imagine, not every division problem can be done so quickly. For example, it is true that 4 divides evenly into 972 (as you'll soon see), but how many times?

To discover the answer, we'll need to learn a process called the **Long Division Algorithm**, or just the **Division Algorithm**. (An *algorithm* is a set of repeated rules that leads to a desired result.) This algorithm works even if there is a remainder (other than 0).

| **The Long Division Algorithm** | **Examples** |
|---|---|
| 1. If possible, divide the divisor into the first (left-most) digit in the dividend, whether or not it divides in evenly.<br>If the first digit of the dividend is too small, divide the divisor into the first two or more (left-most) digits. Place the quotient over the last digit used. | 978 ÷ 6 259 ÷ 7<br> |
| 2. Multiply the quotient and the divisor; place this product directly under the digits used in this division. |  |
| 3. Subtract. |  |
| 4. "Bring down" the first unused digit in the dividend, and repeat this process (starting at step 1) until you can divide no further. Write the remainder next to the quotient.<br>The division process stops when the quotient "covers" the last digit in the dividend. The **remainder** is the amount left over after the last digit in the dividend is covered by the quotient |  |

The example that follows shows the steps one at a time. The explanation requires a lot of space, but your actual problems won't be as long.

**Example 5:** Use the Long Division Algorithm to divide 972 by 4.

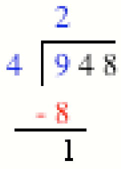
**Answer:**

Steps ① and ②

Recognize that 4 will divide into 9 *two times.*

Place the 2 above the 9 and multiply: 2 x 4 = 8

Subtract this product from 9.



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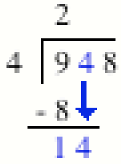
Step ③

Bring down the first unused digit, 4. Start the division process over with a "new" dividend, 14.



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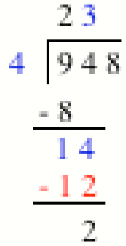
Repeat steps ① and ②

Recognize that 4 will divide into 14 *three times.*

Place the 3 above the 4 (the last digit used) and multiply: 3 x 4 = 12.

Subtract this product from 14.



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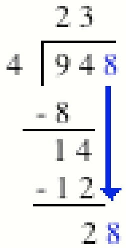
Step ③

Bring down the unused digit, 8. Start the division process over with a "new" dividend, 28.



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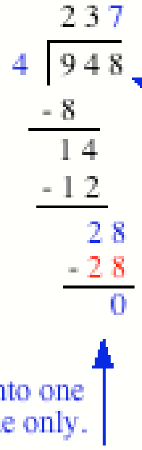
Repeat steps ① and ②

4 divides evenly into 28 *seven times.*

Place the 7 above the 8 (the last digit used) and multiply: 7 x 4 = 28.

Subtract this product from 28 and get a remainder of 0.



Notice that the last digit of the dividend is "covered" by the 7 in the quotient, so we know that we're finished dividing.

Your work will show all of these step combined into one division problem. It will look like the very last one only.

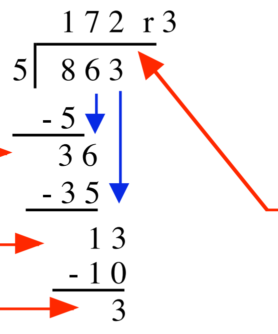
This next example shows all of the steps combined into one division problem. There is some explanation for each step, but please realize that the numbers shown don't appear all at the same time.

**Example 6:** Use the long division algorithm to divide 863 ÷ 5. (You may want to cover up part of the problem so you can see the progress step by step.)

**Answer:**

1. 5 divides into 8 *one* time; place the 1 above the 8 and multiply, 1 x 5 = 5.

   Subtract this product from 8 and bring down the 6.
2. 5 divides into 36 *seven* times; place the 7 above the 6 and multiply, 7 x 5 = 35. Subtract and bring down the 3.
3. 5 divides into 13 *two* times; place the 2 above the 3 and multiply, 2 x 5 = 10. Subtract. There is a remainder of 3.



Continue dividing until the quotient covers the last digit in the dividend.

Now the remainder can be included.

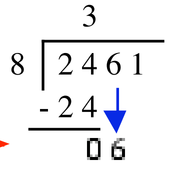
We know that we're finished dividing because the last digit in the dividend is covered by the quotient.

The next example shows what to do when a 0 appears in the quotient.

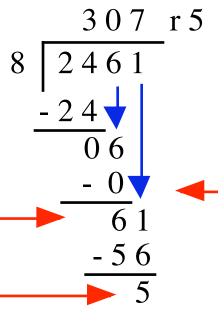
**Example 7:** Use the long division algorithm to divide 2,461 ÷ 8.

**Answer:**

1. 8 won't divide into 2, but it does divide **evenly** into 24 three times; place the 3 above the 4 and multiply, 3 x 8 = 24. Subtracting gives 0, and bring down the 6.



2. At this point, the new dividend is 6 (same as 06), but it is less than 8, and 8 can't divide into 6. So, we say that 8 divides into 6 zero times and place the 0 above the 6.
3. We can multiply the 0 and the 6, but we'll get just 0 and the remainder will still be 6. We then bring down the 1.
4. 8 divides into 61 seven times; place the 7 above the 1 and multiply, 7 x 8 = 56. Subtract. The remainder is 5.



Showing the multiplication by 0 is not necessary.

Instead, once we get a 0 in the quotient, we can just bring down the next digit in the dividend.

Again, in each division problem, we know that we're finished dividing when the last digit in the dividend is covered by the quotient.

$$\begin{array}{rl} & \text{quotient r remainder} \\ \text{divisor} & \overline{\big)\,\text{dividend}} \end{array}$$

**YTI #5** Divide each using long division. (Use the long division symbols set up for you.) Use Examples 5, 6, and 7 as guides.

a) 372 ÷ 4 b) 1,628 ÷ 7 c) 7,835 ÷ 6 d) 40,016 ÷ 8

**Think about it #1:** Why, when using long division, is there subtraction but no addition?

______________________________________________________________________

______________________________________________________________________

### WHEN THE DIVISOR IS A TWO-DIGIT NUMBER

So far in the long division process, all of the divisors have been single-digit numbers. When the divisor contains more than one digit, then we need to do some estimation and, at times, some trial and error.

For example, when dividing 1,167 ÷ 38, we'll first set it up as $38\,\overline{\big)\,1167}$ and prepare to use long division. We know that 38 won't divide into 1, and 38 won't divide into 11. Here, we'll need to use the first *three* digits in the dividend before we can even start to divide. In other words, we'll try to divide 38 into 116.

This can prove to be a challenge by itself. But, if we round each number (the divisor 38 and the three digits 116) to the nearest ten, then we can make an educated guess as to what the first digit of the quotient will be.

> We can estimate that 38 rounds up to 40 and 116 rounds up to 120, so we might think of this as $40\,\overline{\big)\,120}$ . This is, when ignoring the 0's, like dividing $4\,\overline{\big)\,12}$ : 12 ÷ 4 = 3.

This suggests that our choice for the first digit of the quotient should be 3.

**Caution:** Keep an eraser handy because often you'll need to make a second educated guess. Also, the rounded numbers, 40 and 120, are used only for the purpose of making an educated guess. We do not use them in any other part of the division process (though we might need to round 38 again later on in the problem).

**Example 8:** Use the long division algorithm to divide 1,167 by 38.

**Answer:**

$$\begin{array}{r} {}^{+2}\phantom{0} \\ 38 \\ \times\ 3 \\ \hline 114 \end{array}$$

1. 38 won't divide into 1 or 11, but it does divide into 116. After rounding to the nearest ten, we'll make an educated guess that it divides in three times; place the 3 above the 6 and multiply, 3 x 38 = 114. Subtracting gives 2, bring down the 7.

$$\begin{array}{rl} & \phantom{1\,}\phantom{1\,}3\,0 \ \text{ r } 27 \\ 38 & \overline{)\,1\,1\,6\,7} \\ & -1\,1\,4\phantom{\,7}\downarrow \\ \hline & \phantom{1\,1\,}2\,7 \end{array}$$

2. At this point, the new dividend is 27, but it is less than 38. So, 38 divides into 27 zero times and place the 0 above the 7.

**Caution:** The 0 above the 7 is necessary because without it, the quotient wouldn't cover the last digit in the dividend.

Sometime the estimation will get us close but not quite right. If we had chosen to start this quotient...

with 4 we'd get:

$$\begin{array}{r} {}^{+3}\phantom{0} \\ 38 \\ \times\ 4 \\ \hline 152 \end{array} \longrightarrow \begin{array}{rl} & \phantom{1\,1\,}4 \\ 38 & \overline{)\,1\,1\,6\,7} \\ & -1\,5\,2\phantom{\,7} \\ \hline \end{array}$$

Clearly, 152 is too much, so we should start with a quotient value smaller than 4.

and with 2 we'd get:

$$\begin{array}{r} {}^{+1}\phantom{0} \\ 38 \\ \times\ 2 \\ \hline 76 \end{array} \longrightarrow \begin{array}{rl} & \phantom{1\,1\,}2 \\ 38 & \overline{)\,1\,1\,6\,7} \\ & -\phantom{1\,}7\,6\phantom{\,7} \\ \hline & \phantom{1\,}4\,0\phantom{\,7} \end{array}$$

Here the remainder, 40, is larger than the divisor, 38. This means that 38 will divide into 116 at least one more time, so we should start with a quotient value larger than 2.

**YTI #6** Divide each using long division. (Use the long division symbols set up for you.) Use Example 8 as a guide.

a) 936 ÷ 18 b) 8,912 ÷ 32 c) 12,728 ÷ 43

# You Try It Answers

**YTI #1:**
a) dividend: 28 divisor: 7 quotient: 4
b) dividend: 40 divisor: 8 quotient: 5
c) dividend: 12 divisor: 2 quotient: 6

**YTI #2:** a) 10 b) 6 c) 1 d) 8 e) 0 f) 7

**YTI #3:** a) 7 b) 9 c) 1 d) 6

**YTI #4:** a) 8 r 4 b) 1 r 7 c) 7 d) 6 r 4

**YTI #5:** a) 93 b) 232 r 4 c) 1,305 r 5 d) 5,002

**YTI #6:** a) 52 b) 278 r 16 c) 296

## Focus Exercises (Answers are below each, starting at #3.)

***Think again.***

**1.** Why, when using long division, is there subtraction but no addition? *(Think about it #1.)*

**2.** 14 divides evenly in to 266: 266 ÷ 14 = 19. Does this mean that 19 also divides evenly into 266? Explain your answer.

*Divide. Check by multiplying the divisor and the quotient.*

**3.** 20 ÷ 4

**5**

**4.** 28 ÷ 7

**4**

**5.** 0 ÷ 6

**0**

**6.** 0 ÷ 5

**0**

**7.** $\frac{36}{9}$

**4**

**8.** $\frac{24}{8}$

**3**

**9.** $\frac{54}{9}$

**6**

**10.** $\frac{56}{7}$

**8**

**11.** 27 ÷ 3

**9**

**12.** 32 ÷ 4

**8**

**13.** $\frac{35}{5}$

**7**

**14.** $\frac{18}{6}$

**3**

**15.** 80 ÷ 8

**10**

**16.** 90 ÷ 9

**10**

**17.** 40 ÷ 10

**4**

**18.** 50 ÷ 10

**5**

*Divide using long division.*

**19.** 90 ÷ 6

**15**

**20.** 85 ÷ 5

**17**

**21.** 87 ÷ 3

**29**

**22.** 76 ÷ 4

**19**

**23.** $\frac{74}{2}$

**37**

**24.** $\frac{96}{8}$

**12**

**25.** $\frac{91}{7}$

**13**

**26.** $\frac{68}{4}$

**17**

**27.** 115 ÷ 9

**12 r 7**

**28.** 137 ÷ 6

**22 r 5**

**29.** 166 ÷ 5

**33 r 1**

**30.** 183 ÷ 4

**45 r 3**

**31.** $\frac{951}{8}$

**118 r 7**

**32.** $\frac{966}{7}$

**138**

**33.** 1,218 ÷ 4

**304 r 2**

**34.** 2,516 ÷ 5

**503 r 1**

**35.** 18,029 ÷ 3

**6,009 r 2**

**36.** 16,509 ÷ 8

**2,063 r 5**

**37.** 27,036 ÷ 9

**3004**

**38.** 35,042 ÷ 7

**5006**

**39.** 78,300 ÷ 6

**13,050**

**40.** 90,080 ÷ 4

**22,520**

**41.** $\frac{345}{15}$

**23**

**42.** $\frac{594}{18}$

**33**

**43.** 876 ÷ 12

**73**

**44.** 2,756 ÷ 13

**212**

**45.** 24,054 ÷ 24

**1002 r 6**

**46.** 40,016 ÷ 32

**1,250 r 16**

**47.** 1,386 ÷ 33

**42**

**48.** 1,431 ÷ 27

**53**

**49.** 6,300 ÷ 75

**84**

**50.** 8,645 ÷ 91

**95**

**51.** 258,387 ÷ 129

**2,003**

**52.** 375,250 ÷ 125

**3,002**

**53.** 209,100 ÷ 204

**1,025**

**54.** 625,770 ÷ 306

**2,045**

***Think Outside the Box.***

*Evaluate each expression.*

**55.** (345 + 510) ÷ 19

**45**

**56.** 910 ÷ (17 + 9)

**35**

**57.** (1,025 – 349) ÷ 13

**52**

**58.** 901 ÷ (51 – 34)

**53**

**59.** (20 · 51) ÷ 17

**60**

**60.** 972 ÷ (3 · 9)

**36**

**61.** (896 ÷ 4) ÷ 7

**32**

**62.** 817 ÷ (133 ÷ 7)

**43**