Section 0.5 Common Factors and Multiples

Contents:

Thus far, we have looked at individual factors of a number as well as the prime factorization of a number. The next thing for us to consider is the **common factors** of two numbers.

The **common factors** of two numbers are all of the numbers that are factors of both numbers. Another way to this is "The **common factors** of two numbers are all of the numbers that divide evenly into both numbers."

To best understand what this means, we'll start with an example

Example 1: Find the common factors of 24 and 36

Procedure: Consider all of the factors of both 24 and 36, and make a list of factors that they have in common.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Answer: Factors that are common to both 24 and 36: 1, 2, 3, 4, 6, 12

Every one of the numbers in the list divides evenly into both 24 and 36.

Common factors can be *extracted* from the two numbers using a division method that may be a little different from what you already know about division.

To extract a common factor of two numbers, 24 and 36, set the numbers next to each other (with a space between them) and draw an "upside down long division symbol," as shown below. If a number divides evenly into *both* numbers, write it off to the left side and show the *quotient* of each below the original numbers (as shown).

We'd start by writing just $\begin{array}{|cc} 24 & 36 \\ \hline \end{array}$ (the two numbers with the upside down long-division symbol)

Then, find a number that is a factor of (divides evenly into) both numbers, such as 2:

$$\begin{array}{r|cc} 2 & 24 & 36 \\ \hline & 12 & 18 \end{array}$$

There are, of course, other numbers that divide evenly into both 24 and 36, so we could show

$$\begin{array}{r|cc} 3 & 24 & 36 \\ \hline & 8 & 12 \end{array} \qquad \begin{array}{r|cc} 4 & 24 & 36 \\ \hline & 6 & 9 \end{array} \qquad \begin{array}{r|cc} 6 & 24 & 36 \\ \hline & 4 & 6 \end{array} \qquad \begin{array}{r|cc} 12 & 24 & 36 \\ \hline & 2 & 3 \end{array}$$

Each of these **extracted factors** is both a *divisor* of 24 and 36 and a *factor* of 24 and 36.

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THE GREATEST COMMON FACTOR (GCF)

Clearly, the highest, or *greatest*, of the common factors of 24 and 36 is 12; so, 12 is the **greatest common factor (GCF)** of 24 and 36.

Oftentimes, in algebra, we are most interested in the *greatest* common factor only. However, the GCF doesn't always show itself so easily. There is a technique, called the **Division Method**, that will allow us to repeatedly find (and *extract*) common factors. This next example—and these guidelines—illustrates the Division Method for finding the GCF.

Guidelines for Finding the Greatest Common Factor

1. Recognize an obvious common factor (besides 1; 1 is a common factor to every pair of numbers).
2. Divide both numbers by that common factor—now called a **divisor**—to get two **quotients** (the numbers that result from dividing).
3. See if the quotients, themselves, have a common factor; if they do, divide these quotients by it to get even newer quotients. Repeat this process until there aren't any more common prime factors among the new quotients.
4. Make a list of all of the *common* factors found (the divisors on the left). The GCF will result when you multiply all of these divisors together.

Example 2: Find common factors of 60 and 140.

Procedure: Follow the steps outlined above. We'll use an "upside-down" division symbol to assist us in the process.

Answer: Because they both end in 0, they have 10 as a common factor.

Extract 10: Divide both 60 and 140 by 10, and write the quotients below.

$$\begin{array}{r|cc} \mathbf{10} & 60 & 140 \\ \hline & 6 & 14 \end{array} \qquad \begin{array}{l} \Leftarrow \text{original numbers} \\ \Leftarrow \text{quotients (new numbers)} \end{array}$$

Now find a common prime factor between 6 and 14; since they are even, 2 is a common factor; repeat the process:

Extract 2: Divide both 6 and 14 by 2, and write the quotients below.

$$\begin{array}{r|cc} \mathbf{2} & 6 & 14 \\ \hline & 3 & 7 \end{array} \qquad \begin{array}{l} \Leftarrow \text{divide again} \\ \Leftarrow 2^{\underline{nd}} \text{ pair of quotients} \end{array}$$

Since the second pair of quotients, 3 and 7, have no common factor, this part is done. We next write a list of all of the divisors (the numbers on the left side) and multiply them to find the GCF:

(The divisors—the common factors—are 10 and 2.)

So, the **GCF** of 60 and 140 is $\mathbf{10 \cdot 2 =} \boxed{\mathbf{20}}$

Example 3 is exactly the same as Example 2 but with the process shown as you might write it out.

Example 3: Find common factors of 60 and 140 (again).

Procedure: Here is the process for finding common factors in its entirety, from beginning to end.

Answer:

$$\begin{array}{r|rr} \mathbf{10} & 60 & 140 \\ \hline \mathbf{2} & 6 & 14 \\ \hline & 3 & 7 \end{array}$$

⇐ divide by 10

⇐ divide by 2 this time

⇐ we cannot divide this time

So, the **GCF** of 60 and 140 is $\mathbf{10 \cdot 2 =} \boxed{\mathbf{20}}$

Exercise 1: Use the division method to find the *greatest* common factor (GCF) of the following pairs of numbers. Use Example 3 as a guide.

a) the GCF of 8 and 36

The GCF of 8 and 36 is __________

b) the GCF of 60 and 90

The GCF of 60 and 90 is ________

c) the GCF of 15 and 24

The GCF of 15 and 24 is __________

d) the GCF of 20 and 28

The GCF of 20 and 28 is ________

Exercise 2: Find the *greatest* common factor (GCF) of the following pairs of numbers.

a) the GCF of 12 and 36

The GCF of 12 and 36 is ________

c) the GCF of 20 and 50

The GCF of 20 and 50 is ________

e) the GCF of 24 and 44

The GCF of 24 and 44 is ________

b) the GCF of 40 and 96

The GCF of 40 and 96 is ________

d) the GCF of 28 and 42

The GCF of 28 and 42 is ________

f) the GCF of 36 and 54

The GCF of 36 and 54 is ________

Exercise 3: Find the greatest common factor of the following pairs of numbers.

a) the GCF of 45 and 60

The GCF of 45 and 60 is ________

c) the GCF of 8 and 40

The GCF of 8 and 40 is ________

e) the GCF of 80 and 120

The GCF of 80 and 120 is ________

b) the GCF of 60 and 96

The GCF of 60 and 96 is ________

d) the GCF of 28 and 49

The GCF of 28 and 49 is ________

f) the GCF of 42 and 54

The GCF of 42 and 54 is ________

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RELATIVELY PRIME

What if the two numbers have no apparent common factors? Not to worry, they still have 1 as a common factor. Consider the following:

Since 1 is a factor of every number, every pair of numbers will have at least one factor in common, namely 1. So, if two numbers have no other factors in common, then their greatest common factor is 1.

We have a special name for a pair of numbers whose only factor is 1. We say that the numbers are **relatively prime**.

> When two numbers have no other common factors,
> we say that the two numbers are **relatively prime**.
> In this case, the **GCF** is **1**.

The strange thing is, it's quite possible for two *composite* numbers to be relatively prime.

Even though two numbers, such as 14 and 15, are composite numbers, if they have no common factor—other than 1—then we say that they are prime compared to (or relative to) each other; in other words, they are **relatively prime**.

Example 4:	List the common factors of 12 and 35.
Answer:	The factors of 12 are 1, 2, 3, 4, 6, and 12.
	The factors of 35 are 1, 5, 7, and 35.
	Therefore, 12 and 35 have only **1** as a common factor. They are **relatively prime**, and their GCF is 1.

Exercise 4: For each pair of numbers, identify the GCF. See if you can do these in your head using the factor rules for 2, 3 and 5. If they have no common factors, then the GCF is 1, and write "relatively prime" underneath the numbers.

a) 12 and 25

The GCF is ________

b) 8 and 11

The GCF is ________

Exercise 5: For each pair of numbers, identify the GCF. See if you can do these in your head using the factor rules for 2, 3 and 5. If they have no common factors, then the GCF is 1, and write "relatively prime" underneath the numbers.

a) 6 and 14

The GCF is ______

b) 15 and 28

The GCF is ______

c) 12 and 35

The GCF is ______

d) 15 and 25

The GCF is ______

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THE GCF OF THREE NUMBERS

The greatest common factor of three numbers is found very similarly. This time, we're looking for factors (prime or composite) that will divide evenly into all *three* numbers, instead of just two at a time.

Example 5: Find the greatest common factor of 4, 6 and 10.

Procedure: Since they are all even numbers, they all have a factor of 2; so we'll first divide by 2:

Answer:

$$\begin{array}{r|ccc} \mathbf{2} & 4 & 6 & 10 \\ \hline & 2 & 3 & 5 \end{array}$$

⇐ divide by **2**

⇐ we can no longer divide.

The greatest common factor is the *only* common prime factor: **GCF = 2**

Example 5 wasn't too big of a challenge. There was only one common factor. This next example shows that, even though two of the numbers might have a common factor, there might not be even one single prime number that is a factor of all three.

Example 6: Find the greatest common factor of 4, 6 and 9.

Answer: 4 and 6 have a GCF of 2; 6 and 9 have a GCF of 3. However, there is no number, besides 1, that is a common factor of all three numbers, so:

The *only* common factor of 4, 6 and 9: **GCF = 1**

We could say that these *three* numbers (taken all together) are relatively prime.

Example 7: Find the greatest common factor of 24, 36 and 60.

Procedure: All three numbers are even, so we'll first divide by 2:

Answer:

$$\begin{array}{r|lll} \mathbf{2} & 24 & 36 & 60 \\ \hline \mathbf{2} & 12 & 18 & 30 \\ \hline \mathbf{3} & 6 & 9 & 15 \\ \hline & 2 & 3 & 5 \end{array}$$

⇐ divide by **2**

⇐ divide by **2** again

⇐ divide by **3** this time

⇐ we can no longer divide.

The greatest common factor is still found by multiplying the common factors that were found:

GCF = 2 · 2 · 3 = 12 The greatest common factor is 12.

It's possible that you can see the GCF between three number without going through this division process, and that's okay (in fact, it's great!). However, this division process allows us to keep our work organized for when the GCF isn't as obvious.

Exercise 6: Find the greatest common factor of the following pairs of numbers.

a) the GCF of 15, 18 and 33

The GCF is ____________

b) the GCF of 12, 45 and 50

The GCF is ____________

c) the GCF of 15, 45 and 75

The GCF is ____________

d) the GCF of 36, 54 and 90

The GCF is ____________

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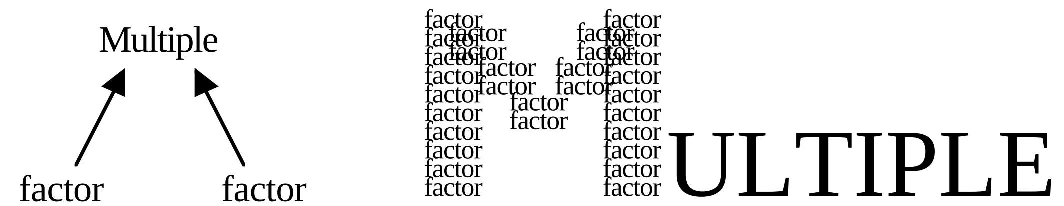
THE LEAST COMMON MULTIPLE

Throughout this section you'll need to remember this phrase:

Factors are the building blocks, and multiples are the building.



Common Multiples and the LCM

Consider the multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60 ... and the list goes on and on. Let's also look at the prime factorization of each of these numbers:

$$\begin{aligned}
6 &= \mathbf{2} \cdot \mathbf{3} \\
12 &= 2 \cdot \mathbf{2} \cdot \mathbf{3} \\
18 &= \mathbf{2} \cdot \mathbf{3} \cdot 3 \\
24 &= 2 \cdot 2 \cdot \mathbf{2} \cdot \mathbf{3} \\
30 &= \mathbf{2} \cdot \mathbf{3} \cdot 5 \\
36 &= 2 \cdot \mathbf{2} \cdot \mathbf{3} \cdot 3 \\
42 &= \mathbf{2} \cdot \mathbf{3} \cdot 7 \\
48 &= 2 \cdot 2 \cdot 2 \cdot \mathbf{2} \cdot \mathbf{3} \\
54 &= \mathbf{2} \cdot \mathbf{3} \cdot 3 \cdot 3 \\
60 &= 2 \cdot \mathbf{2} \cdot \mathbf{3} \cdot 5
\end{aligned}$$

Do you see the factors of 6 ($\mathbf{2} \cdot \mathbf{3}$) in each of these multiple of 6?

In one place or another, each multiple of 6 contains *all* of the factors of 6.

Now consider the multiples of 15: 15, 30, 45, 60, 75, 90, 105, 120 ... and the list goes on and on. Let's also look at the prime factorization of each of these numbers:

$$\begin{aligned}
15 &= \mathbf{3} \cdot \mathbf{5} \\
30 &= 2 \cdot \mathbf{3} \cdot \mathbf{5} \\
45 &= 3 \cdot \mathbf{3} \cdot \mathbf{5} \\
60 &= 2 \cdot 2 \cdot \mathbf{3} \cdot \mathbf{5} \\
75 &= \mathbf{3} \cdot \mathbf{5} \cdot 5 \\
90 &= 2 \cdot 3 \cdot \mathbf{3} \cdot \mathbf{5} \\
105 &= \mathbf{3} \cdot \mathbf{5} \cdot 7 \\
120 &= 2 \cdot 2 \cdot 2 \cdot \mathbf{3} \cdot \mathbf{5}
\end{aligned}$$

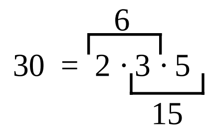
Do you see the factors of 15 ($\mathbf{3} \cdot \mathbf{5}$) in each of factors of 15?

In one place or another, each multiple of 15 contains *all* of the factors of 15.

From those two lists of multiples, we can find two common multiples of 6 and 15; they are **30** and **60**. If we were to continue the list of multiples of 6, we'd find that 6 and 15 also have **90** and **120** as common multiples. In fact, the list of common multiples is endless.

What's true, though, about each of the common multiples of 6 and 15 (the 30, 60, 90, 120, etc.) is that they each, individually, contain all of the factors of 6 and all of the factors of 15.

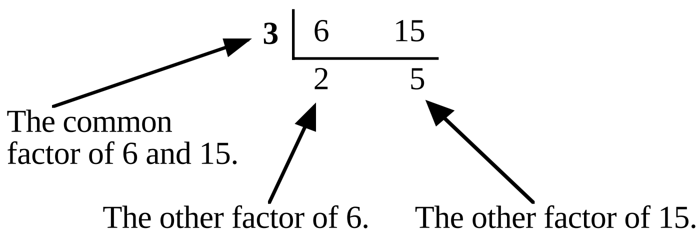
Take **30**, for example, the first common multiple of 6 and 15. In its prime factorization form, you can see the prime factorizations of both 6 and 15:



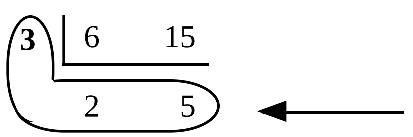
Notice also that they are sharing the common factor of 3. Sure, both 6 and 15 have a factor of 3, but it doesn't need to be represented twice; the factors of 2 and 3 represent the prime factors of 6, and the factors of 5 and 3 (the same 3) represent the prime factors of 15.

Again, we could say that the common multiples of 6 and 15 are 30, 60, 90, 120 ... , and each of them will have both 6 $(2 \cdot 3)$ and 15 $(3 \cdot 5)$ as factors, but the **least common multiple (LCM)** of 6 and 15 is 30. (This is sometimes referred to as the ***lowest*** **common multiple**.)

Look what happens when we try to find that common factor (3) that 6 and 15 share; the division method shows this:



When we extract the common factor from 6 and 15, we are left with other factor of 6 (which is 2) and the other factor of 15 (which is 5). All together, the common factor, 3, and the other factors, 2 and 5, make up the least common multiple (LCM) = $3 \cdot 2 \cdot 5 = 30$.



This is in the shape of an **L**, which stands for LCM.

The point is this, once we have extracted *all* of the common factors, the remaining "quotients" are also factors of the LCM.

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FINDING THE LCM: THE DIVISION METHOD

Example 8: Find the <u>Least</u> Common Multiple (LCM) of:

a) 12 and 30 b) 18 and 90 c) 60 and 70 d) 4 and 15

Answer: Start by using the division method for the GCF.

a) 12 and 30:

$$\begin{array}{r|cc} 2 & 12 & 30 \\ \hline 3 & 6 & 15 \\ \hline & \mathbf{2} & \mathbf{5} \end{array}$$

⇐ divide by **2**
⇐ divide by **3**
⇐ **the last pair of quotients**

For the LCM, we use more than just the factors along the side; we also use the last pair of quotients:

LCM = (side factors) · (last quotients)

LCM = (2 · 3) · (2 · 5) = $\boxed{\mathbf{60}}$

b) 18 and 90:

$$\begin{array}{r|cc} 2 & 18 & 90 \\ \hline 3 & 9 & 45 \\ \hline 3 & 3 & 15 \\ \hline & \mathbf{1} & \mathbf{5} \end{array}$$

⇐ divide by **2**
⇐ divide by **3**
⇐ divide by **3** again
⇐ the last pair of quotients

LCM = (2 · 3 · 3) · (1 · 5) = $\boxed{\mathbf{90}}$

c) 60 and 70:

$$\begin{array}{r|cc} 10 & 60 & 70 \\ \hline & \mathbf{6} & \mathbf{7} \end{array}$$

⇐ divide by **10** (a composite factor)
⇐ the last pair of quotients

LCM = 10 · (6 · 7) = $\boxed{\mathbf{420}}$

d) 4 and 15:

$$\begin{array}{r|cc} 1 & 4 & 15 \\ \hline & \mathbf{4} & \mathbf{15} \end{array}$$

⇐ 1 is the only common factor
⇐ the only pair of quotients (4 and 15 are relatively prime.)

LCM = 1 · (4 · 15) = $\boxed{\mathbf{60}}$

Exercise 7: Find the Least Common Multiple (**LCM**) of the following pairs of numbers. Use Examples 6 and 7 as guides.

a) 8 and 6

LCM = ________________ = $\boxed{}$
(multiply the factors)

b) 12 and 9

LCM = ________________ = $\boxed{}$
(multiply the factors)

c) 25 and 30

LCM = ________________ = $\boxed{}$
(multiply the factors)

d) 18 and 30

LCM = ________________ = $\boxed{}$
(multiply the factors)

e) 9 and 10

LCM = ________________ = $\boxed{}$
(multiply the factors)

f) 6 and 25

LCM = ________________ = $\boxed{}$
(multiply the factors)

Exercise 8: Find the Least Common Multiple (**LCM**) of the following pairs of numbers. Use Examples 5 and 6 as guides.

a) 30 and 45

LCM = ______________ (multiply the factors) = $\boxed{}$

b) 15 and 75

LCM = ______________ (multiply the factors) = $\boxed{}$

c) 80 and 120

LCM = ______________ (multiply the factors) = $\boxed{}$

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Finding the LCM of Three Numbers

The division method for finding the LCM, presented earlier in this section, works well—as you know— for two numbers. It will also work for three numbers but it has a slightly different twist to it.

You have already used the division method to find the GCF of three numbers. The method presented here will *not* lead to the GCF, only the LCM of three numbers.

The major distinction is that we can either look for common factors of all three numbers or we can look for common factors of just two of the numbers. In either case, we divide out the common factors as before.

This is easier to explain using an example. Let’s find the GCF of 18, 30 and 50.

Example 9: Find the least common multiple of 18, 30 and 50.

Procedure: All three numbers are even, so we'll first divide by 2:

Answer:

$$\begin{array}{r|ccc} \mathbf{2} & 18 & 30 & 50 \\ \hline & 9 & 15 & 25 \\ \hline \end{array}$$ ⇐ divide by **2**

At this point, there is no common factor of all three numbers. However, 9 and 15 have a common factor of 3. If we are to use this "special edition" division method correctly, we need to divide out *all* common factors, even if they are common between only two of the numbers.

You might be thinking, "Well 15 and 25 have a common factor of 5, so why aren't we dividing by 5, instead?"

We will divide by 5, but in the next step. Let's first divide by 3. (It doesn't really matter which you choose to divide by first.)

The twist on all of this is that 25 is not divisible by 3, so what do we do with it? Actually, when using this method, and in this case, dividing by 3 doesn't effect 25, so it remains 25.

$$\begin{array}{r|ccc} \mathbf{3} & 9 & 15 & 25 \\ \hline & 3 & 5 & 25 \end{array}$$ ⇐ divide by **3** this time; 25 is unaffected

Now we can divide the 5 and 25 by 5; this time the 3 is unaffected.

$$\begin{array}{r|ccc} \mathbf{5} & 3 & 5 & 25 \\ \hline & 3 & 1 & 5 \end{array}$$ ⇐ divide by **5** this time; 3 is unaffected

⇐ we can no longer divide.

Shown in one step, it looks like this:

$$\begin{array}{r|ccc} \mathbf{2} & 18 & 30 & 50 \\ \hline \mathbf{3} & 9 & 15 & 25 \\ \hline \mathbf{5} & 3 & 5 & 25 \\ \hline & 3 & 1 & 5 \end{array}$$

⇐ divide by **2**
⇐ divide by **3** this time; 25 is unaffected
⇐ divide by **5** this time; 3 is unaffected
⇐ we can no longer divide.

At this point, there are no common factors (besides 1), so to find the LCM we multiply all of the common factors—the divisors—(2, 3 and 5) along with the last quotients (3, 1 and 5) to get:

LCM = $\underline{(\mathbf{2} \cdot \mathbf{3} \cdot \mathbf{5}) \cdot (3 \cdot 1 \cdot 5) = 450}$

Exercise 9: Find the LCM (least common multiple) of the following pairs of numbers by using the *division* method. Use Example 9 as a guide.

a) 4, 6 and 9

b) 6, 10 and 12

The LCM = =

The LCM = =

c) 6, 15 and 18

d) 12, 18 and 45

The LCM = =

The LCM = =

e) 14, 21 and 28

f) 10, 15 and 40

The LCM = =

The LCM = =

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Answers to each Exercise

Section 0.5

Exercise 1	a)	4	b)	30	c)	3	d)	4
Exercise 2	a)	12	b)	8	c)	10	d)	14
	e)	4	f)	18				
Exercise 3	a)	15	b)	12	c)	8	d)	7
	e)	40	f)	6				
Exercise 4	a)	relatively prime			b)	relatively prime		
Exercise 5	a)	2	b)	rel. prime	c)	rel. prime	d)	5
Exercise 6	a)	3	b)	rel. prime	c)	15	d)	18
Exercise 7	a)	24	b)	36	c)	150	d)	90
	e)	90	f)	150				
Exercise 8	a)	90	b)	75	c)	240		
Exercise 9	a)	36	b)	60	c)	90	d)	180
	e)	84	f)	120				

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Section 0.5 Focus Exercises

1. Find the GCF of each pair of numbers. Use any method.

a) 15 and 30

b) 36 and 48

The GCF of 15 and 30 is ________

The GCF of 36 and 48 is ______

c) 45 and 75

d) 40 and 72

The GCF of 45 and 75 is ________

The GCF of 40 and 72 is ______

e) 90 and 210

f) 42 and 154

The GCF of 90 and 210 is ________

The GCF of 42 and 154 is ______

2. Find the least common multiple LCM of each pair of numbers. Uses any method.

a) 8 and 18

b) 15 and 30

The LCM of 8 and 18 is ________

The LCM of 15 and 30 is ________

c) 36 and 48

d) 45 and 54

The LCM of 36 and 48 is ________

The LCM of 45 and 54 is ________

3. Find the least common multiple LCM of each set of numbers.

a) 4, 6 and 15

b) 8, 12 and 20

The LCM of 4, 6 and 15 is ________

The LCM of 8, 12 and 20 is ________

c) 14, 21 and 35

d) 9, 15 and 25

The LCM of 14, 21 and 35 is ________

The LCM of 9, 15 and 25 is ________

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