Section 0.6 Introduction to Fractions

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VOCABULARY OF FRACTIONS

The number on *top* of the fraction bar is called the **numerator**, and the number on the *bottom* is called the **denominator**.

Remember this: $\dfrac{\textbf{numerator}}{\textbf{denominator}}$.

Most fractions that you are going to work with will have a whole number in both the numerator and the denominator. First, let's classify the types of fractions you will see.

1. A **proper fraction**'s numerator is *less* than the denominator, such as $\frac{1}{7}$, $\frac{2}{3}$, $\frac{6}{11}$ and $\frac{23}{24}$.
2. An **improper fraction** has a numerator that is *equal to* or *greater* than the denominator, such as $\frac{7}{7}$, $\frac{5}{3}$, $\frac{30}{11}$ and $\frac{24}{24}$.
3. A **complex fraction** is a fraction in which either the numerator or denominator (or both) is, itself, a fraction. such as $\dfrac{\frac{7}{5}}{2}$, $\dfrac{1}{\frac{2}{3}}$, $\dfrac{\frac{2}{5}}{\frac{3}{4}}$.

The **reciprocal** of a fraction, $\frac{A}{B}$, is another fraction, $\frac{B}{A}$, as long as $A \neq 0$.

In algebra, it's rare for us to use mixed numbers, like $6\frac{1}{2}$ or $3\frac{7}{8}$. Instead, it is much more common to leave improper fractions as they are and not rewrite them as mixed numbers. In algebra, fractions like $\frac{8}{3}$, $\frac{15}{2}$, and $\frac{31}{4}$ are fine to leave as they are.

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A FRACTION AS DIVISION

A fraction is another form of division. The **fraction bar** is another way of expressing division. For example, $\frac{8}{2}$ can also be written $8 \div 2$. Sometimes the *fraction bar* is referred to as the *division bar*.

Also, division can be rewritten in terms of multiplication. This is demonstrated in the "circular relationship" between division and multiplication, on the next page.

We really cannot discuss division without multiplication; <u>the two operations are inverses of each other</u>. When we say, "14 ÷ 2" we also mean, "How many *times* will 2 go into 14?" we're suggesting an undeniable relationship between division and multiplication:

$$14 \div 2 = 7 \quad \text{because} \quad 2 \cdot 7 = 14$$

In a fractional form we can see a "circular" relationship:

$$\frac{14}{2} = 7 \quad \Rightarrow \quad \frac{14}{2} = 7 \quad \Rightarrow \quad 2 \times 7 = 14$$

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UNDEFINED VALUES

Applying the basic operations always result in a number of one kind or another. That is, except, when we divide by 0; dividing by 0 (zero) results in an **undefined value**.

Based on the circular relationship between multiplication and division, dividing by zero leads to a situation for which there is no result. We might ask the question, "14 divided by 0 equals what?" We'd never get a satisfactory answer.

$$\frac{14}{0} = ? \quad \Rightarrow \quad \frac{14}{0} = ? \quad \Rightarrow \quad 0 \times ? = 14$$

Of course, since $0 \cdot A = 0$, no matter the value of A, the product of 0 and any other number will never be 14, so there is no satisfactory result, no defined value. We say, therefore, that $\frac{14}{0}$ is **undefined**, and that <u>a denominator may never be 0</u>. We write this as,

Rule for Division by Zero

If A stands for Any number, then $\mathbf{A \div 0}$, or $\mathbf{\frac{A}{0}}$, is ***undefined***.

This is true for every value of A. Another way to express this rule is:

the denominator of a fraction can never be zero.

Can the numerator ever be zero? Let's take a look at this question using the circular relationship.

$$\frac{0}{6} = ? \quad \Rightarrow \quad \frac{0}{6} = ? \quad \Rightarrow \quad 6 \times ? = 0$$

In this case, the question mark is 0; in other words, $6 \cdot \mathbf{0} = 0$. Therefore, $\frac{0}{6} = 0$.

So, in short:

$$\boxed{\text{If } \mathbf{A} \text{ is any number (except 0), then } \frac{\mathbf{0}}{\mathbf{A}} = \mathbf{0}.}$$

Think about it Why does it say "except 0" in the box above?

Answer: Because, if A can be 0, then 0 would be in the denominator of the fraction $\frac{0}{A}$, giving $\frac{0}{0}$, which is undefined.

Can a fraction that has 0 in the numerator, like $\frac{0}{3}$, have a reciprocal? Why or why not?

Answer: No. A number like $\frac{0}{3}$ cannot have a reciprocal because the reciprocal, $\frac{3}{0}$, would have 0 in the denominator.

Exercise 1: For each, identify the value; if there is no value, write "undefined."

a) $\frac{0}{6} =$ b) $\frac{5}{0} =$ c) $\frac{0}{9} =$

d) $\frac{1}{0} =$ e) $\frac{0}{1} =$ f) $\frac{0}{0} =$

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THE FIRST RULES OF FRACTIONS

To get started, let's look at some of the rules involved in manipulating fractions.

THE FIRST RULES OF FRACTIONS

Rule 1:	$\frac{a}{a} = 1$	Any number (except 0) divided by itself is 1.
Rule 2:	$\frac{a}{1} = a$	Any number divided by 1 is the number itself.
Rule 3:	$\frac{a}{b} \cdot 1 = \frac{a}{b}$	Multiplying by 1 doesn't change the value. (This is true for any number, not just fractions.)
Rule 4:	$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$	Rule of multiplication of fractions

Example 1: Multiply these fractions using Rule #4.

a) $\frac{5}{7} \cdot \frac{2}{3} = \frac{5 \cdot 2}{7 \cdot 3} = \frac{10}{21}$

b) $\frac{3}{2} \cdot \frac{5}{4} = \frac{3 \cdot 5}{2 \cdot 4} = \frac{15}{8}$

c) $\frac{2}{5} \cdot \frac{3}{3} = \frac{2 \cdot 3}{5 \cdot 3} = \frac{6}{15}$

d) $\frac{7}{7} \cdot \frac{8}{3} = \frac{7 \cdot 8}{7 \cdot 3} = \frac{56}{21}$

Exercise 2: Multiply these fractions using Rule #4.

a) $\frac{7}{6} \cdot \frac{5}{3} =$

b) $\frac{8}{3} \cdot \frac{2}{7} =$

c) $\frac{1}{4} \cdot \frac{3}{10} =$

d) $\frac{5}{8} \cdot \frac{9}{1} =$

e) $\frac{1}{6} \cdot \frac{5}{5} =$

f) $\frac{4}{4} \cdot \frac{2}{1} =$

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EQUIVALENT FRACTIONS

Two fractions that have the same value—even though they may look different—are considered to be **equivalent fractions**.

There are two ways to create equivalent fractions:

(1) We can **build up** a fraction (using a combination of Rules 1 and 3) by multiplying it by 1.

For example, $\frac{1}{2}$ can be multiplied by 1 (Rule 3) so as not to change its value; we can make 1 look like $\frac{3}{3}$ (Rule 1) if we want, so we get

$$\frac{1}{2} \cdot 1 = \frac{1}{2} \cdot \frac{3}{3} = \frac{3}{6}$$

So, $\frac{1}{2}$ is equivalent to $\frac{3}{6}$ (even though they look different).

(2) We can **simplify** or **reduce** a fraction by dividing both numerator and denominator by the same value, their greatest common factor (GCF).

For example, $\frac{3}{6}$ can be *reduced by a factor of 3* because the GCF of 3 and 6 is 3;

$$\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}.$$

So, $\frac{3}{6}$ is equivalent to $\frac{1}{2}$ (even though they look different).

Here is the rule used to simplify fractions:

Rule 5: $\frac{a}{b} = \frac{a \div c}{b \div c}$ If we divide the numerator and denominator by a common factor, c, we are able to maintain equivalent fractions.

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We'll look at simplifying fractions in a little bit, but first let's look at .....

BUILDING UP FRACTIONS

We can build up any fraction to an equivalent fraction just by multiplying by 1. As we've seen, 1 can be any fraction of the form $\frac{a}{a}$.

Let's practice building up one fraction to get an equivalent fraction. This means that both the numerator and the denominator become larger—are built up—but the result is an equivalent fraction.

Example 2: Multiply. Notice that the second fraction, in each case, is equivalent to 1.

a) $\frac{5}{4} \cdot \frac{2}{2} = \underline{\frac{10}{8}}$ b) $\frac{9}{7} \cdot \frac{4}{4} = \underline{\frac{36}{28}}$

Exercise 3 Multiply. Notice that the second fraction, in each case, is equivalent to 1. Use Example 2 as a guide.

a) $\frac{5}{6} \cdot \frac{3}{3} =$ ______ b) $\frac{7}{4} \cdot \frac{5}{5} =$ ______ c) $\frac{1}{2} \cdot \frac{6}{6} =$ ______

Next, we can begin to build up a fraction so that it has a specific denominator. For example, if you want $\frac{2}{3}$ to be built up to a fraction that has a denominator of 12, you need to find the right fraction of **1** to multiply by $\frac{2}{3}$.

Example 3: Build up $\frac{2}{3}$ so that it has a denominator of 12.

start with		multiply by **1**		to get
$\frac{2}{3}$	$\cdot$	$\frac{?}{?}$	$=$	$\frac{?}{12}$

In this case, **1** should be written as $\frac{4}{4}$:

$\rightarrow$ $\frac{2}{3} \cdot \frac{4}{4} = \frac{8}{12}$

So, $\underline{\frac{2}{3} \text{ is equivalent to } \frac{8}{12}}$.

Exercise 4: Multiply by the appropriate value of 1 to build up each fraction to one with a denominator as shown. Use Example 3 as a guide.

start with	multiply by 1	to get		start with	multiply by 1	to get
a) $\frac{2}{5}$	$\cdot\ \frac{}{}$	$= \frac{}{35}$		b) $\frac{8}{9}$	$\cdot\ \frac{}{}$	$= \frac{}{45}$
c) $\frac{9}{7}$	$\cdot\ \frac{}{}$	$= \frac{}{28}$		d) $\frac{7}{6}$	$\cdot\ \frac{}{}$	$= \frac{}{60}$

Example 4: Build up the fraction $\frac{2}{3}$ to have the same denominator as $\frac{5}{6}$.

Procedure: We must multiply $\frac{2}{3}$ by 1 so that it becomes an equivalent fraction with a denominator of 6. Use $\frac{2}{2}$ for 1: $\frac{2}{3} \cdot \frac{2}{2} = \frac{4}{6}$

Exercise 5: Build up the first fraction so that it has the same denominator as the second fraction.

a) Build up $\frac{3}{5}$ to have the same denominator as $\frac{7}{20}$:

b) Build up $\frac{2}{9}$ to have the same denominator as $\frac{5}{18}$:

c) Build up $\frac{7}{10}$ to have the same denominator as $\frac{19}{60}$:

d) Build up $\frac{4}{15}$ to have the same denominator as $\frac{7}{45}$:

We can also build up each of two fractions so that they both have the same (new) denominator.

Example 5: Build up the fractions $\frac{1}{4}$ and $\frac{5}{6}$ to have the same denominator of 12.

Answer: We must multiply each fraction by 1, but the fraction we use for 1 doesn't need to be the same.

For $\frac{1}{4}$, use $\frac{3}{3}$ for 1: For $\frac{5}{6}$, use $\frac{2}{2}$ for 1:

$\frac{1}{4} \cdot \frac{3}{3} = \frac{3}{\mathbf{12}}$ $\frac{5}{6} \cdot \frac{2}{2} = \frac{10}{\mathbf{12}}$

Exercise 6: Build up, individually, the given fractions so that they have the same denominator. Use Example 5 as a guide.

a) Build up both $\frac{1}{4}$ and $\frac{6}{7}$ to have the same denominator of 28.

b) Build up both $\frac{5}{8}$ and $\frac{1}{6}$ to have the same denominator 24.

c) Build up both $\frac{4}{9}$ and $\frac{7}{12}$ to have the same denominator of 36.

d) Build up both $\frac{11}{15}$ and $\frac{7}{20}$ to have the same denominator of 60.

e) Build up both $\frac{3}{10}$ and $\frac{11}{15}$ to have the same denominator of 30.

f) Build up both $\frac{7}{10}$ and $\frac{7}{12}$ to have the same denominator of 60.

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SIMPLIFYING (REDUCING) FRACTIONS

It is important to be able to recognize that some fractions *look* different from one another but actually have the same value—they are *equivalent* fractions.

> For example, a social worker is asked to conduct a survey among her clients and write a report on her findings. Since her case load is so large, she decides to ask the first 12 clients that come in to see her on Monday morning.
>
> From the survey she finds that 8 of her clients have new jobs. In her report, she wants to state that $\frac{8}{12}$ of her clients have new jobs, but her supervisor says she must show it as a reduced fraction. What fraction should she use in her report?

Here, again, is Rule 5:

Rule 5:	$\frac{a}{b} = \frac{a \div c}{b \div c}$	If we divide the numerator and denominator by a common factor, c, we are able to maintain equivalent fractions.

In the situation with the social worker, she had to simplify the fraction $\frac{8}{12}$. Let's see the steps necessary to simplify this fraction.

1. First consider the GCF of the numerator and denominator; the GCF of 8 and 12 is $2 \cdot 2 = \mathbf{4}$.

$$\begin{array}{r|rr} 2 & 8 & 12 \\ \hline 2 & 4 & 6 \\ \hline & 2 & 3 \end{array}$$

2. Divide the numerator and the denominator by the GCF 4: $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$

You may also divide this mentally;

$$\frac{\cancel{8}^{\,2}}{{}_{3}\cancel{12}} = \frac{2}{3}$$

In either case, we can say that $\frac{8}{12}$ has been ***reduced by a factor of 4*** to $\frac{2}{3}$.

It is also appropriate to say that $\frac{8}{12}$ and $\frac{2}{3}$ are **equivalent fractions**; they have the same value.

Example 6: Simplify each of the following fractions by dividing both the numerator and denominator by a common factor.

a) $\frac{7}{21}$ b) $\frac{30}{5}$

Answer: Identify the greatest common factor of both the numerator and the denominator, and divide each by that number. The division shown here could be done mentally.

a) $\frac{7}{21} = \frac{7 \div 7}{21 \div 7} = \boxed{\frac{1}{3}}$ $\frac{7}{21}$ has been reduced by a factor of 7.

b) $\frac{30}{5} = \frac{30 \div 5}{5 \div 5} = \frac{6}{1} = \boxed{6}$ (by Rule 2)

$\frac{30}{5}$ has been reduced, by a factor of 5, to a whole number.

Sometimes, in simplifying fractions, the common factor we choose to reduce the fraction doesn't simplify it completely, so we might need to reduce the fraction further.

Example 7: Reduce $\frac{24}{60}$ by a common factor.

Answer: There are many common factors of 24 and 60; One such factor is 3, though it is not the greatest common factor. Let's see what happens.

$\frac{24}{60} = \frac{24 \div 3}{60 \div 3} = \frac{8}{20}$

$\frac{8}{20}$ is **equivalent** to $\frac{24}{60}$, but $\frac{8}{20}$ can be simplified further by a factor of 4.

$\frac{8}{20} = \frac{8 \div 4}{20 \div 4} = \boxed{\frac{2}{5}}$

In the example above, $\frac{8}{20}$ is a simpler form (a reduced form) of $\frac{24}{60}$. However, $\frac{8}{20}$ can, itself, be reduced. When it is finally reduced to $\frac{2}{5}$ we say that the fraction is ***completely reduced***.

A fraction should always be simplified completely, not just part way. If you simplify a fraction by one factor, you should check to see if the new (equivalent) fraction can, itself, be simplified.

Exercise 7 Simplify completely. Decide what factor should be used to reduce the fraction. Show your work as you simplify. For some of these, you may need to reduce the fraction further. (You may use "mental division" by crossing out and dividing by the GCF.)

a) $\frac{3}{12} =$

b) $\frac{8}{18} =$

c) $\frac{9}{24} =$

d) $\frac{10}{25} =$

e) $\frac{14}{35} =$

f) $\frac{22}{77} =$

g) $\frac{20}{45} =$

h) $\frac{32}{66} =$

i) $\frac{15}{3} =$

j) $\frac{18}{2} =$

k) $\frac{10}{60} =$

l) $\frac{9}{27} =$

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MULTIPLYING FRACTIONS

Here, again, is the rule for the multiplication of fractions.

Rule 4: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

Sometimes, when multiplying fractions, the result is a fraction that can be simplified. Example 8 shows how this can be done.

Example 8: Find the product of the two fractions. Simplify your answer (completely), if possible.

a) $\frac{2}{5} \cdot \frac{3}{7}$ b) $\frac{5}{6} \cdot \frac{4}{15}$ c) $\frac{9}{10} \cdot \frac{4}{3}$ d) $\frac{3}{9} \cdot \frac{6}{4}$

Answer: Apply Rule 4 to each of these; then, if the product (the result) can be simplified, please do so. If it cannot simplify, then leave it as it is.

a) $\frac{2}{5} \cdot \frac{3}{7} = \frac{2 \cdot 3}{5 \cdot 7} = \boxed{\frac{6}{35}}$

This is complete because 6 and 35 are *relatively prime and* have no common factors.

b) $\frac{5}{6} \cdot \frac{4}{15} = \frac{5 \cdot 4}{6 \cdot 15} = \frac{20}{90} = \frac{20 \div 10}{90 \div 10} = \boxed{\frac{2}{9}}$

This fraction ↑ can be simplified by a factor of 10.

c) $\frac{9}{10} \cdot \frac{4}{3} = \frac{9 \cdot 4}{10 \cdot 3} = \frac{36}{30} = \frac{36 \div 6}{30 \div 6} = \boxed{\frac{6}{5}}$

This fraction ↑ can be simplified by a factor of 6.

d) $\frac{3}{9} \cdot \frac{6}{4} = \frac{3 \cdot 6}{9 \cdot 4} = \frac{18}{36} = \frac{18 \div 18}{36 \div 18} = \boxed{\frac{1}{2}}$

This fraction ↑ can be simplified by a factor of 18.

Actually, $\frac{18}{36}$ can be reduced by a factor of 2, a factor of 3, of 6, of 9, or of 18 (the GCF).

If you use a common factor other than 18, you'll need to reduce further in order to **simplify completely**.

Exercise 8: Find the product of the two fractions. Simplify your answer completely. Use Example 8 as a guide.

a) $\frac{6}{7} \cdot \frac{2}{3}$

b) $\frac{8}{11} \cdot \frac{3}{4}$

c) $\frac{10}{9} \cdot \frac{3}{5}$

d) $\frac{3}{9} \cdot \frac{10}{5}$

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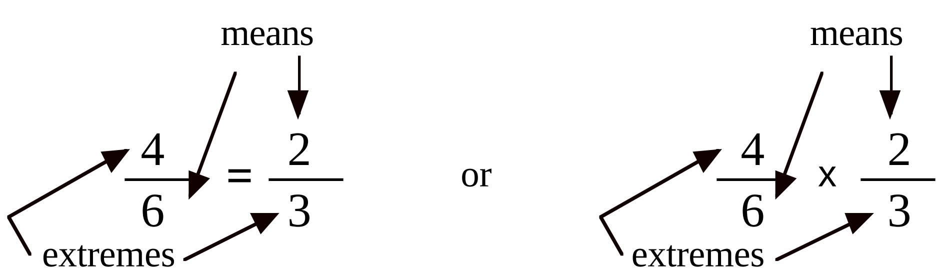
CROSS DIVIDING

The Extremes and Means of a Product

When two fractions are either equivalent to each or are being multiplied we have names for the *positions* of the numerators and denominators. We have these names only so that we may talk about the positions more easily. In particular, one pair of positions is called the **extremes** and the other pair of positions is called the **means**. These definitions are very visual, so here is a diagram.



In Equality **In Multiplication**

When considering the **extremes** and **means**, the actual values of the numbers aren't as important as the *positions* of the numbers.

Cross Dividing

In the multiplication $\mathbf{\frac{2}{3}} \cdot \mathbf{\frac{5}{7}}$, **2** and **7** are the **extremes** and **3** and **5** are the **means**.

The process in reducing the fractions in multiplication *before* multiplying them together is referred to as *cross divide*ing, because we are able to divide out common factors *across* the multiplication sign. We can use **cross dividing** if either the means or the extremes (or both) have a common factor.

Here are some of the same examples seen earlier in this section. This time, though, the common factors will be divided out before multiplying.

Example 9: Find the product of the two fractions by first looking for common factors between the means and extremes.

a) $\frac{2}{5} \cdot \frac{3}{7}$ b) $\frac{5}{6} \cdot \frac{7}{15}$ c) $\frac{9}{10} \cdot \frac{4}{3}$ d) $\frac{9}{27} \cdot \frac{10}{4}$

Answer: Be sure to first look for common factors between the numerators and the denominators.

a) $\frac{2}{5} \cdot \frac{3}{7}$ There are **no** common factors between the means or between the extremes, so we just multiply as we have before. Therefore,

$$\frac{2}{5} \cdot \frac{3}{7} = \frac{2 \cdot 3}{5 \cdot 7} = \boxed{\frac{6}{35}}$$

Notice that this fraction cannot reduce.

b) $\frac{5}{6} \cdot \frac{7}{15}$ There is a common factor of 5 between the **extremes**, 5 and 15, so

$$\frac{\overset{1}{\cancel{5}}}{6} \cdot \frac{7}{\underset{3}{\cancel{15}}} = \frac{1}{6} \cdot \frac{7}{3} = \boxed{\frac{7}{18}}$$

c) $\frac{9}{10} \cdot \frac{4}{3}$ There is a common factor of 3 between the **extremes**, 9 and 3; there is also a common factor of 2 between the **means**, 10 and 4, so

$$\frac{\overset{3}{\cancel{9}}}{\underset{5}{\cancel{10}}} \cdot \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{3}}} = \frac{3}{5} \cdot \frac{2}{1} = \boxed{\frac{6}{5}}$$

d) $\frac{9}{27} \cdot \frac{10}{4}$ In this case, we cannot “cross divide” because the neither the means nor the extremes have a common factor. However, each fraction can be reduced individually.

$$\frac{\overset{1}{\cancel{9}}}{\underset{3}{\cancel{27}}} \cdot \frac{\overset{5}{\cancel{10}}}{\underset{2}{\cancel{4}}} = \frac{1}{3} \cdot \frac{5}{2} = \boxed{\frac{5}{6}}$$

Exercise 9: Find the product of the two fractions. Simplify your answer completely. Use Example 9 as a guide.

a) $\frac{6}{7} \cdot \frac{2}{3}$ b) $\frac{8}{11} \cdot \frac{3}{4}$

c) $\frac{10}{9} \cdot \frac{3}{5}$ d) $\frac{3}{9} \cdot \frac{10}{5}$

What if one of the "fractions" we're multiplying is really a whole number? Then we need only write the whole number as a fraction: $\frac{\textbf{whole number}}{\mathbf{1}}$.

Example 10: Multiply by first writing the whole number as a fraction (with a denominator of 1).

a) $8 \cdot \frac{7}{4}$ b) $\frac{5}{9} \cdot 12$

Answer: a) $8 \cdot \frac{7}{4} = \frac{8}{1} \cdot \frac{7}{4} = \frac{2}{1} \cdot \frac{7}{1} = \frac{14}{1} = \boxed{14}$

We can cross divide ↑ reducing the **extremes** by a factor of 4.

b) $\frac{5}{9} \cdot 12 = \frac{5}{9} \cdot \frac{12}{1} = \frac{5}{3} \cdot \frac{4}{1} = \boxed{\frac{20}{3}}$

We can cross divide ↑ reducing the **means** by a factor of 3.

Exercise 10: Find the product. First, rewrite the whole number as a fraction. Simplify your answer completely. Use Example 10 as a guide.

a) $4 \cdot \frac{2}{3}$ b) $\frac{5}{8} \cdot 4$

c) $7 \cdot \frac{3}{4}$ d) $\frac{7}{6} \cdot 9$

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DIVIDING FRACTIONS

The **reciprocal** of a fraction, $\frac{A}{B}$, is another fraction, $\frac{B}{A}$, as long as $A \neq 0$.

Furthermore, when finding the reciprocal of a fraction, we say that the fraction has been **inverted**.

When the fraction $\frac{A}{B}$ is **inverted**, it is another fraction, $\frac{B}{A}$, as long as $A \neq 0$.

Basically, the **reciprocal** of a fraction is the same as the **inverse** of the fraction.

In general, if both numbers are already fractions we get Rule 6:

Rule 6: Division of fractions

$$\frac{a}{b} \div \frac{d}{c} = \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

When dividing by a fraction, we use **"invert and multiply**:**"**

- keep the first fraction the same,
- write the multiplication sign, and
- invert (write the reciprocal of) the second fraction.

Before we simplify by cross-canceling, the problem must *first* be changed to multiplication.

Example 11: Divide. Using Rule 6, invert and multiply. Simplify if possible.

a) $\frac{4}{7} \div \frac{3}{5}$ b) $\frac{6}{7} \div \frac{5}{6}$

Answer: Notice the factors that divide out, either by cross dividing or within the same fraction. They are not highlighted, here.

a) $\frac{4}{7} \div \frac{3}{5}$ **(Invert and multiply)**

$= \frac{4}{7} \cdot \frac{5}{3} = \boxed{\frac{20}{21}}$

b) $\frac{6}{7} \div \frac{5}{6}$ (The 6's can**NOT** divide out.)

$= \frac{6}{7} \cdot \frac{6}{5} = \boxed{\frac{36}{35}}$

Example 12: Divide. Using Rule 6, invert and multiply. Simplify if possible.

a) $10 \div 5$ b) $3 \div 4$ c) $6 \div \frac{3}{5}$ d) $\frac{2}{3} \div 2$

e) $\frac{7}{4} \div \frac{1}{3}$ f) $\frac{9}{4} \div \frac{15}{8}$ g) $12 \div \frac{9}{5}$ h) $\frac{4}{3} \div \frac{5}{4}$

Answer: You may use cross dividing if you wish, but it is not shown here. Also, you may NOT cross divide within *division*!

a) $10 \div 5$

$$= \frac{10}{1} \cdot \frac{1}{5} = \frac{10}{5} = \boxed{2}$$

b) $3 \div 4$

$$= \frac{3}{1} \cdot \frac{1}{4} = \boxed{\frac{3}{4}}$$

c) $6 \div \frac{3}{5}$

$$= \frac{6}{1} \cdot \frac{5}{3} = \frac{30}{3} = \boxed{10}$$

d) $\frac{2}{3} \div 2$

$$= \frac{2}{3} \cdot \frac{1}{2} = \frac{2}{6} = \boxed{\frac{1}{3}}$$

e) $\frac{7}{4} \div \frac{1}{3}$

$$= \frac{7}{4} \cdot \frac{3}{1} = \boxed{\frac{21}{4}}$$

f) $\frac{4}{3} \div \frac{5}{4}$

$$= \frac{4}{3} \cdot \frac{4}{5} = \boxed{\frac{16}{15}}$$

g) $12 \div \frac{9}{5}$

$$= \frac{12}{1} \cdot \frac{5}{9}$$

$$= \frac{4 \cdot 3}{1} \cdot \frac{5 \cdot 1}{3 \cdot 3}$$

$$= \frac{4}{1} \cdot \frac{5}{3} = \boxed{\frac{20}{3}}$$

h) $\frac{9}{4} \div \frac{15}{8}$

$$= \frac{9}{4} \cdot \frac{8}{15}$$

$$= \frac{3 \cdot 3}{4 \cdot 1} \cdot \frac{2 \cdot 4}{3 \cdot 5}$$

$$= \frac{3}{1} \cdot \frac{2}{5} = \boxed{\frac{6}{5}}$$

Exercise 11: Divide. Using Rule 6, invert and multiply. Simplify if possible. Use Example 12 as a guide.

a) $\frac{5}{6} \div \frac{3}{7}$

b) $\frac{4}{3} \div \frac{5}{4}$

c) $\frac{10}{7} \div \frac{2}{3}$

d) $\frac{5}{6} \div \frac{7}{12}$

e) $\frac{8}{5} \div \frac{8}{5}$

f) $\frac{1}{6} \div \frac{1}{3}$

g) $7 \div 3$

h) $12 \div 9$

i) $6 \div \frac{4}{5}$

j) $\frac{10}{9} \div 5$

Example 13: Simplify this *complex* fraction by first rewriting it using the division sign (÷).

$$\frac{\frac{15}{14}}{\frac{20}{21}}$$

Procedure: Rewrite it as a division problem.

Answer:

$$\frac{\frac{15}{14}}{\frac{20}{21}} = \frac{15}{14} \div \frac{20}{21}$$

$$= \frac{15}{14} \cdot \frac{21}{20} = \frac{3 \cdot 5}{2 \cdot 7} \cdot \frac{7 \cdot 3}{5 \cdot 4} = \frac{3}{2} \cdot \frac{3}{4} = \boxed{\frac{9}{8}}$$

Exercise 12: Simplify each complex fraction by first rewriting it using the division sign (÷). Use Example 13 as a guide.

a) $\dfrac{\frac{8}{15}}{\frac{16}{35}}$

b) $\dfrac{\frac{9}{10}}{\frac{27}{25}}$

c) $\dfrac{3}{\frac{5}{2}}$

d) $\dfrac{4}{\frac{8}{7}}$

e) $\dfrac{\frac{8}{9}}{5}$

f) $\dfrac{\frac{15}{25}}{6}$

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Answers to each Exercise

Section 0.6

Exercise 1

a) 0 b) undefined c) 0 d) undefined
e) 0 f) undefined

Exercise 2

a) $\frac{35}{18}$ b) $\frac{16}{21}$ c) $\frac{3}{40}$ d) $\frac{45}{8}$
e) $\frac{5}{30}$ f) $\frac{8}{4}$

Exercise 3

a) $\frac{15}{18}$ b) $\frac{35}{20}$ c) $\frac{6}{12}$

Exercise 4

a) $\frac{2}{5} \cdot \frac{7}{7} = \frac{14}{35}$ b) $\frac{8}{9} \cdot \frac{5}{5} = \frac{40}{45}$ c) $\frac{9}{7} \cdot \frac{4}{4} = \frac{36}{28}$

d) $\frac{7}{6} \cdot \frac{10}{10} = \frac{70}{60}$

Exercise 5

a) $\frac{3}{5} \cdot \frac{4}{4} = \frac{12}{20}$ b) $\frac{2}{9} \cdot \frac{2}{2} = \frac{4}{18}$

c) $\frac{7}{10} \cdot \frac{6}{6} = \frac{42}{60}$ d) $\frac{4}{15} \cdot \frac{3}{3} = \frac{12}{45}$

Exercise 6

a) $\frac{1}{4} \cdot \frac{7}{7} = \frac{7}{28}$ and $\frac{6}{7} \cdot \frac{4}{4} = \frac{24}{28}$

b) $\frac{5}{8} \cdot \frac{3}{3} = \frac{15}{24}$ and $\frac{1}{6} \cdot \frac{4}{4} = \frac{4}{24}$

c) $\frac{4}{9} \cdot \frac{4}{4} = \frac{16}{36}$ and $\frac{7}{12} \cdot \frac{3}{3} = \frac{21}{36}$

d) $\frac{11}{15} \cdot \frac{4}{4} = \frac{44}{60}$ and $\frac{7}{20} \cdot \frac{3}{3} = \frac{21}{60}$

e) $\frac{3}{10} \cdot \frac{3}{3} = \frac{9}{30}$ and $\frac{11}{15} \cdot \frac{2}{2} = \frac{22}{30}$

f) $\frac{7}{10} \cdot \frac{6}{6} = \frac{42}{60}$ and $\frac{7}{12} \cdot \frac{5}{5} = \frac{35}{60}$

Exercise 7 a) $\frac{1}{4}$ b) $\frac{4}{9}$ c) $\frac{3}{8}$ d) $\frac{2}{5}$

e) $\frac{2}{5}$ f) $\frac{2}{7}$ g) $\frac{4}{9}$ h) $\frac{16}{33}$

i) 5 j) 9 k) $\frac{1}{6}$ l) $\frac{1}{3}$

Exercise 8 a) $\frac{4}{7}$ b) $\frac{6}{11}$ c) $\frac{2}{3}$ d) $\frac{2}{3}$

Exercise 9 a) $\frac{4}{7}$ b) $\frac{6}{11}$ c) $\frac{2}{3}$ d) $\frac{2}{3}$

Exercise 10 a) $\frac{8}{3}$ b) $\frac{5}{2}$ c) $\frac{21}{4}$ d) $\frac{21}{2}$

Exercise 11 a) $\frac{35}{18}$ b) $\frac{16}{15}$ c) $\frac{15}{7}$ d) $\frac{10}{7}$

e) 1 f) $\frac{1}{2}$ g) $\frac{7}{3}$ h) $\frac{4}{3}$

i) $\frac{15}{2}$ j) $\frac{2}{9}$

Exercise 12 a) $\frac{7}{6}$ b) $\frac{5}{6}$ c) $\frac{6}{5}$ d) $\frac{7}{2}$

e) $\frac{8}{45}$ f) $\frac{1}{10}$

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Section 0.6 Focus Exercises

1. Multiply by the appropriate value of 1 to build up each fraction to one with a denominator as shown.

	start with	multiply by **1**	to get		start with	multiply by **1**	to get
a)	$\frac{7}{10}$	$\cdot\ \frac{\quad}{\quad}$	$= \frac{\quad}{30}$	b)	$\frac{5}{12}$	$\cdot\ \frac{\quad}{\quad}$	$= \frac{\quad}{24}$
c)	$\frac{4}{9}$	$\cdot\ \frac{\quad}{\quad}$	$= \frac{\quad}{18}$	d)	$\frac{3}{8}$	$\cdot\ \frac{\quad}{\quad}$	$= \frac{\quad}{40}$
e)	$\frac{1}{9}$	$\cdot\ \frac{\quad}{\quad}$	$= \frac{\quad}{72}$	f)	$\frac{6}{1}$	$\cdot\ \frac{\quad}{\quad}$	$= \frac{\quad}{9}$

2. Simplify *completely*. Show your work as you simplify.

a) $\frac{9}{15} =$ b) $\frac{21}{28} =$ c) $\frac{20}{60} =$

d) $\frac{16}{44} =$ e) $\frac{60}{18} =$ f) $\frac{27}{18} =$

g) $\frac{8}{32} =$ h) $\frac{15}{35} =$ i) $\frac{54}{18} =$

3. Find the product of the two fractions. Simplify your answer completely.

a) $\frac{5}{8} \cdot \frac{4}{3}$ b) $\frac{10}{7} \cdot \frac{2}{5}$ c) $\frac{12}{25} \cdot \frac{5}{4}$

d) $\frac{15}{8} \cdot \frac{12}{25}$ e) $\frac{8}{20} \cdot \frac{21}{9}$ f) $\frac{5}{3} \cdot 24$

4. Find the quotient of each pair of fractions. Simplify if possible.

a) $\frac{8}{3} \div \frac{5}{2}$

b) $\frac{9}{5} \div \frac{3}{8}$

c) $\frac{12}{5} \div \frac{4}{15}$

d) $\frac{10}{9} \div \frac{8}{15}$

e) $20 \div \frac{4}{3}$

f) $\frac{30}{7} \div 12$

5. Simplify each complex fraction by first rewriting it using the division sign (÷).

a) $\dfrac{\frac{7}{9}}{\frac{3}{10}}$

b) $\dfrac{\frac{14}{15}}{\frac{21}{40}}$

c) $\dfrac{10}{\frac{15}{4}}$

d) $\dfrac{\frac{20}{9}}{8}$

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