Section 0.7 Adding and Subtracting Fractions

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THE UNIT FRACTION

Definition: A **unit fraction** is one in which the numerator is **1** and the denominator is any non-zero whole number.

Example 1: The following *are* **unit fractions**: $\frac{1}{7}$ $\frac{1}{4}$ $\frac{1}{12}$

The following are **not** unit fractions: $\frac{5}{1}$ $\frac{3}{6}$ 9

$\frac{5}{1}$ is *not* a unit fraction because the numerator is not **1**.

$\frac{3}{6}$ is *not* a unit fraction even though it reduces to one: $\frac{1}{2}$.

9 can be written as $\frac{9}{1}$, but it is *not* a unit fraction.

Exercise 1: Identify which of the following are unit fractions. Circle those that are. Use Example 1 as a guide.

$\frac{3}{1}$ $\frac{2}{8}$ $\frac{1}{6}$ $\frac{10}{15}$ $\frac{1}{8}$ $\frac{1}{2}$

The Rule of Detachment

Every fraction can be written as the product of a whole number and a unit fraction. As a way of explaining this idea, consider Rule 4, the product of two fractions:

Rule 4: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$ This rule can be easily rewritten as $\frac{a \cdot c}{b \cdot d} = \frac{a}{b} \cdot \frac{c}{d}$. In other words, we can **separate** any fraction product. This holds true for a single fraction, as well.

For example, we can write $\frac{3}{5}$ as $\frac{3 \cdot 1}{1 \cdot 5}$ ➔ $\frac{3}{5} = \frac{3 \cdot 1}{1 \cdot 5}$

and then separate this into the product of two fractions: $= \frac{3}{1} \cdot \frac{1}{5}$.

The first fractions can then simplify to a whole number, thereby becoming $= 3 \cdot \frac{1}{5}$.

This leads to the rule of detachment:

Rule of Detachment: $\frac{a}{b} = a \cdot \frac{1}{b}$

Example 2: Use the Rule of Detachment to rewrite each of these fractions as a product of a whole number and a unit fraction.

a) $\frac{3}{7} = 3 \cdot \frac{1}{7}$ b) $\frac{8}{5} = 8 \cdot \frac{1}{5}$ c) $\frac{1}{4} = 1 \cdot \frac{1}{4}$

Exercise 2 Use the Rule of Detachment to rewrite each of these fractions as a product of a whole number and a unit fraction.

a) $\frac{5}{12} =$ b) $\frac{9}{2} =$ c) $\frac{4}{4} =$ d) $\frac{1}{8} =$

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NAMING FRACTIONS

We can take the idea of the unit fraction a little further. For example, $\frac{1}{2}$ is often read as "one half." It is also sometimes read as just "a half." Similarly, $\frac{1}{4}$ is "a fourth."

How might we, then, read $\frac{3}{4}$? You would probably read it as "three-fourths," but let's see how we use the Rule of Detachment along with naming fractions:

$\frac{3}{4} = 3 \cdot \frac{1}{4}$, suggesting that we have **3 fourths**.

We don't need to rewrite a fraction in order to name it. $\mathbf{\frac{5}{8}}$ suggests that we have **5 eighths**.

Example 3: Use the rule of detachment and ***name*** the result.

a) $\frac{7}{8} = \mathbf{7} \cdot \frac{1}{8}$ **7 eighths** b) $\frac{9}{13} = \mathbf{9} \cdot \frac{1}{13}$ **9 thirteenths**

c) $\frac{8}{5} = \mathbf{8} \cdot \frac{1}{5}$ **8 fifths** d) $\frac{1}{4} = \mathbf{1} \cdot \frac{1}{4}$ **1 fourth**

Exercise 3 Use the rule of detachment and *name* the result. Use Example 3 as a guide.

a) $\frac{4}{7}$ = ________ b) $\frac{10}{9}$ = ________

c) $\frac{3}{2}$ = ________ d) $\frac{1}{6}$ = ________

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LIKE UNITS (OF MEASURE) AND LIKE FRACTIONS

Is it possible that 2 + 3 does *not* equal 5? By themselves, no. 2 + 3 = 5 always. However, if units of measure are included, then we need to be a little more careful. Get this:

2 hours + 3 hours = 5 hours,

but 2 hours + 3 minutes ≠ 5 of anything.

When units of measure are involved, we cannot add them directly if the units are not the same. In other words, in order to add units of measure, the units must be "like" each other.

Just as units must be *like* before combining (adding or subtracting), so must fractions.

In other words, two fractions can be added or subtracted as long as they are **like fractions**.

> **Like fractions** are two (or more) fractions that have the same denominator, often called the **common denominator**.

Just as	2 hours	+	3 hours	=	5 hours,
Likewise	2 sevenths	+	3 sevenths	=	5 sevenths.
or	$\frac{2}{7}$	+	$\frac{3}{7}$	=	$\frac{5}{7}$

However, what we really need to know is that $\frac{2}{7} + \frac{3}{7} = \frac{\mathbf{2+3}}{7} = \frac{5}{7}$.

Notice that, just as when adding **hours**, the result is **hours**; likewise, when adding **sevenths**, the result is **sevenths**.

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Combining Like Fractions

To **combine** like fractions means to either **add** them together or to **subtract** one from the other.

Adding and subtracting like fractions:

Rule of Addition: $\frac{A}{B} + \frac{C}{B} = \frac{A + C}{B}$

Rule of Subtraction: $\frac{A}{B} - \frac{C}{B} = \frac{A - C}{B}$

Example 4: Combine these like fractions (add or subtract):

a) $\frac{8}{13} + \frac{2}{13} = \frac{8 + 2}{13} = \frac{10}{13}$

b) $\frac{12}{19} + \frac{6}{19} = \frac{12 + 6}{19} = \frac{18}{19}$

c) $\frac{9}{11} - \frac{3}{11} = \frac{9 - 3}{11} = \frac{6}{11}$

d) $\frac{7}{9} - \frac{5}{9} = \frac{7 - 5}{9} = \frac{2}{9}$

Exercise 4: Combine these like fractions (add or subtract) Use Example 4 as a guide.

a) $\frac{7}{9} + \frac{1}{9} =$

b) $\frac{12}{19} + \frac{5}{19} =$

c) $\frac{10}{13} - \frac{5}{13} =$

d) $\frac{11}{15} - \frac{4}{15} =$

e) $\frac{6}{7} - \frac{5}{7} =$

f) $\frac{14}{25} + \frac{8}{25} =$

You may have noticed that none of these answers simplify. That might not always be the case, though, so let’s look at some that do.

Example 5: Combine these like fractions. Simplify if possible.

a) $\frac{9}{16} + \frac{3}{16} = \frac{9+3}{16} = \boxed{\frac{12}{16}} = \boxed{\frac{3}{4}}$

b) $\frac{6}{8} - \frac{1}{8} = \frac{6-1}{8} = \boxed{\frac{5}{8}}$

c) $\frac{11}{12} - \frac{5}{12} = \frac{11-5}{12} = \boxed{\frac{6}{12}} = \boxed{\frac{1}{2}}$

d) $\frac{7}{11} + \frac{9}{11} = \frac{7+9}{11} = \boxed{\frac{16}{11}}$

e) $\frac{3}{10} + \frac{7}{10} = \frac{3+7}{10} = \boxed{\frac{10}{10}} = \boxed{1}$

f) $\frac{1}{2} - \frac{1}{2} = \frac{1-1}{2} = \boxed{\frac{0}{2}} = \boxed{0}$

Exercise 5: Combine these like fractions. Simplify if possible. Use Example 5 as a guide.

a) $\frac{8}{15} + \frac{2}{15}$

b) $\frac{9}{8} - \frac{5}{8}$

c) $\frac{3}{4} + \frac{5}{4}$

d) $\frac{8}{9} - \frac{7}{9}$

e) $\frac{5}{16} + \frac{11}{16}$

f) $\frac{8}{10} - \frac{1}{10}$

g) $\frac{8}{20} + \frac{2}{20}$

h) $\frac{9}{18} - \frac{5}{18}$

i) $\frac{7}{14} + \frac{9}{14}$

j) $\frac{18}{25} - \frac{3}{25}$

k) $\frac{8}{30} + \frac{12}{30}$

l) $\frac{18}{24} - \frac{4}{24}$

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Combining Fractions with Unlike Denominator

Let's again consider adding 2 hours + 3 minutes = ???

We know that we can't combine these (as yet) because they are not like units. However, we can rewrite 2 hours as 120 minutes, giving

120 minutes + 3 minutes = 123 minutes.

Similarly 2 feet + 3 inches = ???? (can't combine)

but 2 feet is equivalent to 24 inches: 24 inches + 3 inches = 27 inches.

The point is this: as long as we can write things in terms of the same unit of measure, we can combine them. For example, $\frac{1}{2} + \frac{5}{12}$ cannot be combined as they are because they are not like fractions:

1 half + 5 twelfths = ???? (can't combine)

But we know that $\frac{6}{12}$ is equivalent to $\frac{1}{2}$, so we can use $\frac{6}{12}$ in place of $\frac{1}{2}$:

6 twelfths + 5 twelfths = 11 twelfths

In other words, $\frac{6}{12} + \frac{5}{12} = \frac{11}{12}$

We have seen that fractions having the same denominators are called **like fractions**. So, fractions that have different denominators are **unlike fractions**.

In the pervious section we looked at building up two fractions to have the same denominator. This is called finding the **common denominator**. We've got to be careful, though, that we don't pick just *any* denominator.

Example 6: Build up both $\frac{1}{4}$ and $\frac{5}{6}$ so that each has a denominator of 12.

Answer: For $\frac{1}{4}$ we'll multiply by $\frac{3}{3}$: $\frac{1}{4} \cdot \frac{3}{3} = \frac{3}{12}$

For $\frac{5}{6}$ we'll multiply by $\frac{2}{2}$: $\frac{5}{6} \cdot \frac{2}{2} = \frac{10}{12}$

Exercise 6: Follow the instructions. Use Example 6 as a guide.

a) Build up both $\frac{2}{5}$ and $\frac{1}{3}$ so that each has a denominator of 15.

b) Build up both $\frac{3}{8}$ and $\frac{5}{6}$ so that each has a denominator of 24.

Example 7: Build up both $\frac{1}{3}$ and $\frac{7}{9}$ so that each has a denominator of 9.

Procedure: $\frac{7}{9}$ already has the common denominator of 9, so it doesn't need to built up.

Answer: For $\frac{1}{3}$ we'll multiply by $\frac{3}{3}$: $\frac{1}{3} \cdot \frac{3}{3} = \frac{3}{9}$ and $\frac{7}{9}$ stays as it is.

Exercise 7: Follow the instructions. Use Example 7 as a guide.

a) Build up both $\frac{3}{4}$ and $\frac{5}{8}$ so that each has a denominator of 8.

b) Build up both $\frac{4}{5}$ and $\frac{7}{10}$ so that each has a denominator of 10.

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Finding the Least Common Denominator

To combine (add or subtract) two fractions, they must first have common denominators. The most efficient common denominator to create is called the **least common denominator**, or **LCD** for short. The LCD is the *least common multiple* of the two denominators.

For example, to find the LCD of the fractions $\frac{1}{4}$ and $\frac{1}{6}$ we need to first find the least common multiple of the denominators 4 and 6. We'll use a technique from Section 0.5:

$$\begin{array}{r|ll} \mathbf{2} & 4 & 6 \\ \hline & \mathbf{2} & \mathbf{3} \end{array}$$

⇐ divide by 2

⇐ the last pair of quotients

$$\text{LCD} = 2 \cdot (2 \cdot 3) = \boxed{12}$$

Example 8: Find the least common denominator of each pair of fractions, then build them up to have that same denominator.

Procedure: The *least common denominator* (LCD) of $\frac{2}{9}$ and $\frac{1}{6}$ is the *least common multiple* of 9 and 6.

Answer:

$$\begin{array}{r|rr} 3 & 9 & 6 \\ \hline & 3 & 2 \end{array}$$

So the LCD is $3 \cdot (3 \cdot 2) = 18$

So, build up each fraction to have a common denominator of 18:

$$\frac{2}{9} \cdot \frac{2}{2} = \frac{4}{18} \quad \text{and} \quad \frac{1}{6} \cdot \frac{3}{3} = \frac{3}{18}$$

Exercise 8 Find the least common denominator of each pair of fractions, then build them up to have that same denominator.

a) $\frac{3}{5}$ and $\frac{1}{2}$

b) $\frac{1}{4}$ and $\frac{5}{14}$

c) $\frac{5}{6}$ and $\frac{3}{8}$

d) $\frac{4}{9}$ and $\frac{7}{12}$

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ADDING AND SUBTRACTING UNLIKE FRACTIONS

When adding or subtracting unlike fractions, we must first decide on a common denominator that we will build up each fraction to have. Some common denominators are better than others.

As you will see in this next example, the **least common denominator (LCD)** is the best choice. The LCD is the least common multiple (LCM) of the denominators. You may want to refer back to Section 0.5 to remind yourself about the LCM.

Example 9: Evaluate $\frac{1}{6} + \frac{1}{4}$ by first getting *a* common denominator. Completely simplify the result.

Procedure: Really, we can get any common denominator we choose, as long as it is a common multiple of 4 and 6.

Answer: a) Let's choose their product, 24:

$$\frac{1}{6} + \frac{1}{4}$$

$$= \frac{1}{6} \cdot \frac{4}{4} + \frac{1}{4} \cdot \frac{6}{6}$$

$$= \frac{4}{24} + \frac{6}{24} = \frac{4+6}{24} = \boxed{\frac{10}{24}}$$

However, $\frac{10}{24}$ can be reduced by a factor of 2: $\frac{10}{24} = \frac{10 \div 2}{24 \div 2} = \boxed{\frac{5}{12}}$

b) Let's choose the least common multiple of 6 and 4, which is 12:

$$\frac{1}{6} + \frac{1}{4}$$

$$= \frac{1}{6} \cdot \frac{2}{2} + \frac{1}{4} \cdot \frac{3}{3}$$

$$= \frac{2}{12} + \frac{3}{12} = \frac{2+3}{12} = \boxed{\frac{5}{12}}$$

Of course, this is the same answer as in part (a), but by first finding the LCD, we didn't need to reduce the answer.

One point to note: In the previous example, it didn't matter which common denominator we chose, either 24 or 12. We could have chosen even 36 or 48. In the end, if we do everything correctly,

$\frac{1}{4} + \frac{1}{6}$ will eventually (after reducing) turn out to be $\frac{5}{12}$.

Another point to note: Often, if you choose the *least* common denominator, the result won't need to be reduced at the end. Sometimes, though, reducing the result will be necessary; there's no easy way to predict whether you'll need to reduce or not.

Clearly, though, the best common denominator to choose is the least common multiple of the denominators, usually referred to as the **least common denominator**, or just **LCD**.

Example 10: Evaluate $\frac{5}{6} - \frac{3}{4}$ by first choosing a common denominator and then building up each fraction to that denominator.

Answer: $\frac{5}{6} - \frac{3}{4}$ (We already know that the LCD is 12.)

$= \frac{5}{6} \cdot \frac{2}{2} - \frac{3}{4} \cdot \frac{3}{3}$

$= \frac{10}{12} - \frac{9}{12}$

$= \frac{10-9}{12} = \boxed{\frac{1}{12}}$

Exercise 9 Evaluate each sum or difference by first choosing a common denominator and building up the fractions appropriately.

Be sure to simplify each answer completely. (For each, try to find the **least** common denominator.)

Use Examples 9 and 10 as a guide.

a) $\frac{4}{9} + \frac{1}{6}$

b) $\frac{4}{15} + \frac{9}{10}$

c) $\frac{7}{10} + \frac{3}{8}$

d) $\frac{5}{6} - \frac{7}{10}$

Example 11: Evaluate $\frac{9}{8} - \frac{1}{2}$.

Procedure: This time, you could choose 16 as the common multiple, but the least common multiple, the LCD, is 8, itself. If you choose 8, you need to build up only one of the fractions.

Answer:

$$\frac{9}{8} - \frac{1}{2}$$

$$= \frac{9}{8} - \frac{1}{2} \cdot \frac{4}{4}$$

$$= \frac{9}{8} - \frac{4}{8}$$

$$= \frac{9-4}{8} = \boxed{\frac{5}{8}}$$

LCD:

$$\begin{array}{r|ll} 2 & 8 & 2 \\ \hline & 4 & 1 \end{array}$$

LCD $= 2 \cdot (4 \cdot 1) = 8$

Exercise 10 Evaluate each using the techniques learned in this section. Be sure to simplify each answer completely. (For each, try to find the **least** common denominator.)

You may leave your answers as improper fractions.

a) $\frac{1}{6} + \frac{1}{3}$

b) $\frac{3}{10} + \frac{1}{5}$

c) $\frac{7}{12} + \frac{5}{6}$

d) $\frac{5}{9} - \frac{1}{3}$

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Combining Whole Numbers and Fractions

If one of the "fractions" happens to be a whole number, then it could easily be written as a fraction by making the denominator as **1**. From there, the common denominator will just be the denominator of the other fraction.

Example 12: Evaluate (a) $2 + \frac{3}{5}$ and (b) $1 - \frac{7}{9}$.

Procedure: First write the whole number as a fraction with a denominator of 1.

Answer: a) $2 + \frac{3}{5}$

$= \frac{2}{1} + \frac{3}{5}$

$= \frac{2}{1} \cdot \frac{5}{5} + \frac{3}{5}$

$= \frac{10}{5} + \frac{3}{5} = \boxed{\frac{13}{5}}$

b) $1 - \frac{7}{9}$

$= \frac{1}{1} - \frac{7}{9}$

$= \frac{1}{1} \cdot \frac{9}{9} - \frac{7}{9}$

$= \frac{9}{9} - \frac{7}{9} = \boxed{\frac{2}{9}}$

Exercise 11 Evaluate each using the techniques learned in this section. Be sure to simplify each answer completely. (For each, try to find the **least** common denominator.)

(You may leave your answers as improper fractions.)

a) $2 + \frac{1}{4}$

b) $1 - \frac{2}{3}$

c) $3 + \frac{5}{8}$

d) $2 - \frac{2}{5}$

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Answers to each Exercise

Section 0.7

Exercise 1 $\frac{1}{6}$, $\frac{1}{8}$ and $\frac{1}{2}$ are all unit fractions (because their numerators are 1).

Exercise 2 a) $5 \cdot \frac{1}{12}$ b) $9 \cdot \frac{1}{2}$ c) $4 \cdot \frac{1}{4}$ d) $1 \cdot \frac{1}{8}$

Exercise 3: a) $4 \cdot \frac{1}{7}$ which is 4 sevenths. b) $10 \cdot \frac{1}{9}$ which is 10 ninths.

c) $3 \cdot \frac{1}{2}$ which is 3 halves. d) $1 \cdot \frac{1}{6}$ which is 1 sixth.

Exercise 4 a) $\frac{8}{9}$ b) $\frac{17}{19}$ c) $\frac{5}{13}$ d) $\frac{7}{15}$

e) $\frac{1}{7}$ f) $\frac{22}{25}$

Exercise 5 a) $\frac{10}{15} = \frac{2}{3}$ b) $\frac{4}{8} = \frac{1}{2}$ c) $\frac{8}{4} = 2$ d) $\frac{1}{9}$

e) $\frac{16}{16} = 1$ f) $\frac{7}{10}$ g) $\frac{10}{20} = \frac{1}{2}$ h) $\frac{4}{18} = \frac{2}{9}$

i) $\frac{16}{14} = \frac{8}{7}$ j) $\frac{15}{25} = \frac{3}{5}$ k) $\frac{20}{30} = \frac{2}{3}$ l) $\frac{14}{24} = \frac{7}{12}$

Exercise 6 a) $\frac{2}{5} \cdot \frac{3}{3} = \frac{6}{15}$ and $\frac{1}{3} \cdot \frac{5}{5} = \frac{5}{15}$

b) $\frac{3}{8} \cdot \frac{3}{3} = \frac{9}{24}$ and $\frac{5}{6} \cdot \frac{4}{4} = \frac{20}{24}$

Exercise 7 a) $\frac{3}{4} \cdot \frac{2}{2} = \frac{6}{8}$ and $\frac{5}{8}$ stays as it is

b) $\frac{4}{5} \cdot \frac{2}{2} = \frac{8}{10}$ and $\frac{7}{10}$ stays as it is

Exercise 8

a) Common denominator: 10; so, $\frac{3}{5} \cdot \frac{2}{2} = \frac{6}{10}$; $\frac{1}{2} \cdot \frac{5}{5} = \frac{5}{10}$

b) Common denominator: 28; so, $\frac{1}{4} \cdot \frac{7}{7} = \frac{7}{28}$; $\frac{5}{14} \cdot \frac{2}{2} = \frac{10}{28}$

c) Common denominator: 24; so, $\frac{5}{6} \cdot \frac{4}{4} = \frac{20}{24}$; $\frac{3}{8} \cdot \frac{3}{3} = \frac{9}{24}$

d) Common denominator: 36; so, $\frac{4}{9} \cdot \frac{4}{4} = \frac{16}{36}$; $\frac{7}{12} \cdot \frac{3}{3} = \frac{21}{36}$

Exercise 9 a) $\frac{11}{18}$ b) $\frac{7}{6}$ c) $\frac{43}{40}$ d) $\frac{2}{15}$

Exercise 10 a) $\frac{3}{6} = \frac{1}{2}$ b) $\frac{5}{10} = \frac{1}{2}$ c) $\frac{17}{12}$ d) $\frac{2}{9}$

Exercise 11 a) $\frac{9}{4}$ b) $\frac{1}{3}$ c) $\frac{29}{8}$ d) $\frac{8}{5}$

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Section 0.7 Focus Exercises

1. Write each of these as the product of a whole number and a unit fraction using The Rule of Detachment.

a) $\frac{5}{9}$ = ________ b) $\frac{8}{3}$ = ________ c) $\frac{1}{6}$ = ________

2. Combine these like fractions (add or subtract) Simplify the result whenever possible.

a) $\frac{9}{11} - \frac{3}{11}$ b) $\frac{4}{7} + \frac{2}{7}$

c) $\frac{11}{12} - \frac{2}{12}$ d) $\frac{4}{15} + \frac{2}{15}$

e) $\frac{11}{24} + \frac{7}{24}$ f) $\frac{9}{16} - \frac{5}{16}$

3. Evaluate each sum or difference by first choosing a common denominator and building up the fractions appropriately. Be sure to simplify each answer completely.

a) $\frac{5}{8} - \frac{1}{3}$ b) $\frac{7}{12} - \frac{1}{4}$

4. Evaluate each sum or difference by first choosing a common denominator and building up the fractions appropriately. Be sure to simplify each answer completely.

a) $\frac{3}{5} + \frac{3}{10}$

b) $\frac{8}{25} + \frac{3}{10}$

c) $\frac{5}{9} - \frac{1}{6}$

d) $\frac{4}{15} + \frac{11}{20}$

2. Evaluate each sum or difference by first choosing a common denominator and building up the fractions appropriately. Be sure to simplify each answer completely.

a) $4 + \frac{5}{6}$

b) $5 - \frac{1}{8}$

c) $2 + \frac{5}{9}$

d) $1 - \frac{3}{10}$

e) $6 - \frac{2}{3}$

f) $3 + \frac{4}{5}$

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