Section 1.6 Equations

Objectives
In this section, you will learn to:

- Define the term *equation*.
- Apply the Subtraction Property of Equality.
- Apply the Division Property of Equality.
- Solve simple equations.

To successfully complete this section, you need to understand:

- The identities (1.2)
- The Commutative Properties (1.2)
- Adding whole numbers (1.3)
- Subtracting whole numbers (1.3)
- Multiplying whole numbers (1.4)
- Dividing whole numbers (1.5)

INTRODUCTION

To this point we have looked at some application situations that involve only one operation, either addition, subtraction, multiplication, or division. Here is a situation that involves two operations, addition and subtraction.

> Wendy's monthly budget includes $529 for rent, $175 for food, $192 for utilities and transportation, and $125 for entertainment. If she takes home (after taxes and other deductions) $1,238 per month, how much does she have left at the end of the month for savings?

Such situations—those that involve more than one operation—are more common than you might think. This section, and the next, will introduce you to a little algebra and some fairly consistent techniques for solving applications. First, though, you need to become familiar with the notion of *equations*.

EQUATIONS

An **equation** is a mathematical sentence in which one expression = another expression.

We use an equation when we are asked to find a number that is not yet known. In this way, we say that we are seeking an **unknown value**.

The unknown value that we seek is a number that must be represented in the equation. Since we don't yet know what the number is, we use a letter to represent it. For our purposes, we will use the letter **n**. The letter **n** is called a **variable**, and it represents a **n**umber, the unknown value.

A known value, such as 8, is called a **constant**.

Think about it: The word *variable* can be thought of as *vary - able*. How would you explain to a classmate the difference between the words *variable* and *constant*?

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Here are some examples of equations: $n + 3 = 8$

$$18 = 6 \cdot n$$
$$6 + 5 = n$$
$$28 + 38 + 65 + n = 178.$$

Notice that each equation contains an equal sign (=). This is used to indicate that the two "sides" of the equation—the left side and the right side—are **equivalent**, that is, they have equal value. Notice further that the variable, n, can be on either side of the equal sign.

The purpose of writing an equation is to find the value of the variable, the unknown value. Finding the value of the variable means finding a *solution* to the equation. A **solution** to an equation is a number that makes the equation true.

Example 1: For each equation, is 6 the solution?

a) $n + 5 = 11$ b) $12 = 8 + n$ c) $n \cdot 5 = 35$ d) $24 = 4 \cdot n$

Procedure: For each equation, replace n with 6. Evaluate one side and see if it equals the other side. If they are equal, then 6 makes the equation true, and 6 is the solution. If they are not equal, then 6 does *not* make the equation true, and 6 is *not* the solution.

Answer:

	Equation	Does replacing n with 6 make the equation true?
a)	$n + 5 = 11$	
	$6 + 5 = 11$	
	$11 = 11$	Yes. 6 is the solution.
b)	$12 = 8 + n$	
	$12 = 8 + 6$	
	$12 = 14$	No. 6 is not the solution.
c)	$n \cdot 5 = 35$	
	$6 \cdot 5 = 35$	
	$30 = 35$	No. 6 is not the solution.
d)	$24 = 4 \cdot n$	
	$24 = 4 \cdot 6$	
	$24 = 24$	Yes. 6 is the solution.

YTI #1 For each equation, is 5 the solution? Use Example 1 as a guide.

a) $18 = n + 11$ b) $15 + n = 20$ c) $50 = n \cdot 10$ d) $8 \cdot n = 45$

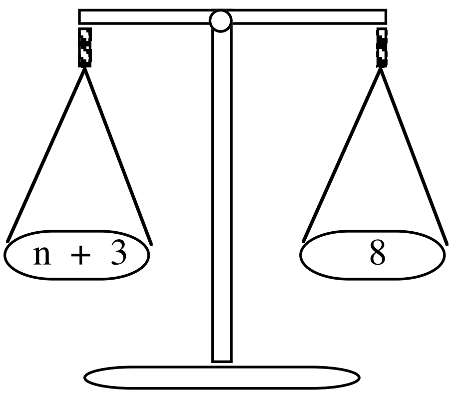
To **solve** an equation means to find the solution, to find the number that makes the equation true.

To do so we must get the variable, n, by itself on one side of the equation and all constants on the other side. This is called **isolating the variable**. The best way to isolate the variable is to remove the constants that are on the same side of the equal sign as n.

For example, in the equation $n + 3 = 8$, we wish to isolate the variable, n, by removing 3 from the left side. This will leave n alone (isolated) on that side.

In the process of isolating the variable, we'll write a new equation based on the original equation. It is important to know that the solution to the new equation must be the same as the solution to the original equation.

Before learning how to isolate the variable, let's explore the idea of a **balanced equation**. All equations must be balanced.

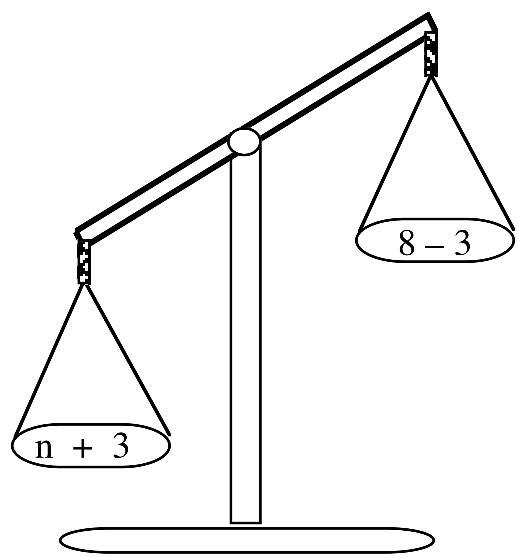


This scale is here to help you visualize a balanced equation.

The equation, $n + 3 = 8$, has a solution of 5. In other words, the equation is true when n is replaced with 5, but it isn't true for any other number.

What do you think would happen if we subtracted some "weight" from just one side of the scale?

Caution:	Subtracting weight from just one side of a balanced scale means *that* side becomes lighter and goes up. We should *not* write a new equation based on what we see (at right) because the sides are not in balance.



So how do we keep an equation in balance?

Whatever value is subtracted from one side must also be subtracted from the other side at the same time; this will keep the equation in balance.

So, we may remove 3, by subtraction from *each* side, to isolate the variable.
Notice that, in subtracting 3 from *each* side, we get a new equation:

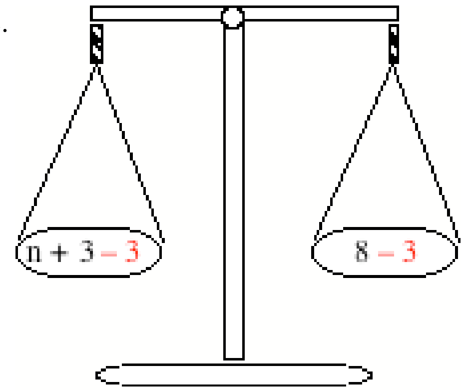
Original equation: $n + 3 = 8$

New equation (subtract 3 from each side): $n + 3 - 3 = 8 - 3$

This simplifies to: $n + 0 = 5$

And this simplifies to just: $n = 5$

The solution is **5**.



Though the solution, itself, is 5, it's often appropriate to leave it as n = 5.

When an equation involves addition, such as $n + 3 = 8$, we can *isolate the variable* using the Subtraction Property of Equality.

The Subtraction Property of Equality

If **n + a = b**, we can isolate the variable by subtracting the same value, **a**, from each side—whether the variable is written first or second—and still have a balanced equation.

	Example A:	**Example B:**
$n + a = b$	$n + 8 = 20$	$18 = 5 + n$
$n + a - a = b - a$	$n + 8 - 8 = 20 - 8$	$18 - 5 = 5 - 5 + n$
↑ a – a is 0, the identity for addition. Adding 0 helps isolate the variable.	$n + 0 = 12$	$13 = 0 + n$
	$n = 12$	$13 = n$
		or **n = 13***
	Check: $12 + 8 = 20$ True!	Check: $18 = 5 + 13$ True!

After solving, we can check the answer to show that it is the solution; replace the variable in the original equation with the answer and see if it makes the equation true.

*Notice that, in Example B, the variable is first isolated on the right side. It is customary to write the final result with n on the left side and the number on the right.

Example 2: Solve the equation $n + 6 = 9$ by using the Subtraction Property of Equality.

Answer: $n + 6 = 9$		To isolate the variable we need to subtract 6 from each side.
	$n + 6 - 6 = 9 - 6$	Apply subtraction to each side.
	$n + 0 = 3$	Because 0 is the identity for addition, $n + 0$ is just n.
	$n = 3$	We now know the value of the variable; it is no longer unknown.

Check: We can show that 3 is the solution by replacing n with 3 in the original equation:

$n + 6 = 9$	becomes
$3 + 6 = 9$	which is true. So, 3 is the **solution**.

Whether the constant is written before or after the variable, we may place the subtraction right after the constant.

Example 3: Solve the equation $15 + n = 23$ by using the Subtraction Property of Equality. Check the answer to show that it is the solution.

Procedure: Isolate the variable by subtracting 15 from each side.

Answer:	$23 = 15 + n$	To isolate the variable we need to subtract 15 from each side.
	$23 - 15 = 15 - 15 + n$	Apply subtraction to each side.
	$8 = 0 + n$	Because 0 is the identity for addition, $0 + n$ is just n.
	$8 = n$	**Check:** Replace n with 8 in the original equation:
Write n on the left:	$n = 8$	$15 + 8 = 23$ which is true. So, the solution is 8.

YTI #2 Solve each equation by using the Subtraction Property of Equality. Show all steps. Use Examples 2 and 3 as guides. Check each answer to show that it is the solution.

a) $n + 8 = 12$ b) $20 = 7 + n$ c) $45 + n = 73$ d) $51 = n + 39$

In some equations, there is more than one constant on the same side as the variable. Before subtracting a constant from each side, we must first add those constants.

Example 4: Solve the equation $15 + 6 + n = 30$.

Procedure: First, simplify the left side by adding $15 + 6$.

Answer:

$15 + 6 + n = 30$ — $15 + 6 = 21$. Rewrite the equation.

$21 + n = 30$ — Now we can subtract 21 from each side.

$21 - 21 + n = 30 - 21$ — Apply subtraction to each side.

$0 + n = 9$ — $0 + n$ is just n.

$n = 9$

Check: Replace n with 9: $15 + 6 + 9 = 30$ ↗

Check:

$$\begin{array}{r} 15 \\ 6 \\ +\ 9 \\ \hline 30 \end{array}\ \checkmark$$

True!

The solution is 9.

YTI #3 Solve the following by isolating the variable. Use Example 4 as a guide.

a) $29 + 61 + n = 96$

b) $113 = n + 46 + 28$

In some equations, the variable is already isolated, but one side needs to be evaluated.

Example 5: Solve the equation $17 + 26 + 58 = n$.

Procedure: Evaluate the left side. Here, the only check necessary is our addition work.

Answer:

$17 + 26 + 58 = n$ — Since n is already isolated (on the right side) we simply need to evaluate the left side:

$101 = n$

$$\begin{array}{r} 17 \\ 26 \\ +\ 58 \\ \hline 101 \end{array}$$

Write n on the left: $n = 101$

YTI #4 Solve the equation. Use Example 5 as a guide.

a) $49 + 57 + 38 = n$

b) $n = 559 + 467$

SOLVING AN EQUATION INVOLVING MULTIPLICATION

When a variable, n, is multiplied by a number, we call that number a **coefficient**, pronounced

"co - ee - fish´ - unt."

Example 6:	In each equation, identify the coefficient.			
	a) $n \cdot 6 = 18$	b) $8 \cdot n = 40$	c) $12 = 4 \cdot n$	
Answer:	a) Coefficient: 6	a) Coefficient: 8	a) Coefficient: 4	

YTI #5 In each equation, identify the coefficient, the number multiplied by the variable. Use Example 6 as a guide.

a) $n \cdot 5 = 30$ Coefficient: ____

b) $7 \cdot n = 28$ Coefficient: ____

c) $24 = 3 \cdot n$ Coefficient: ____

d) $18 = n \cdot 9$ Coefficient: ____

If an equation involves multiplication instead of addition, then we can isolate the variable by *dividing* each side by the same number, the coefficient. This rule is the Division Property of Equality.

The Division Property of Equality

If **n · a = b**, then we can isolate the variable by dividing each side by the same value, **a** (the coefficient)—whether the variable is written first or second—and still have a balanced equation.

	Example C:	**Example D:**
$n \cdot a = b$	$n \cdot 8 = 40$	$36 = 4 \cdot n$
$n \cdot a \div a = b \div a$	$n \cdot 8 \div 8 = 40 \div 8$	$36 \div 4 = 4 \div 4 \cdot n$
↑ $a \div a$ is 1, the identity for multiplication. Multiplying by 1 helps isolate the variable.	$n \cdot 1 = 5$	$9 = 1 \cdot n$
	$n = 5$	$9 = n$
		Write n on the left: $n = 9$

This time, instead of trying to subtract to get 0 (the identity for addition), we want to multiply the variable by 1 (the identity for multiplication).

Example 7: Solve the equation $n \cdot 6 = 18$ using the Division Property of Equality.

Procedure: Isolate the variable by dividing each side by the coefficient, 6.

Answer:

$n \cdot 6 = 18$ — To isolate the variable we need to divide each side by the coefficient 6.

$n \cdot 6 \div 6 = 18 \div 6$ — Apply division to each side.

$n \cdot 1 = 3$ — Because 1 is the identity for multiplication, $n \cdot 1$ becomes just n.

$n = 3$ — We now know the value of the variable; it is no longer unknown.

Check: Replace n with 3 in the original equation:

$3 \cdot 6 = 18$ √ which is true. So, 3 is the **solution**.

Whether the coefficient is written before or after the variable, we may place the division right after the coefficient.

Example 8: Solve the equation $4 \cdot n = 156$ using the Division Property of Equality.

Answer:

$4 \cdot n = 156$ — Divide each side by the coefficient.

$4 \div 4 \cdot n = 156 \div 4$ — Divide each side by 4.

$1 \cdot n = 39$

$n = 39$

So, the solution is 39.

$$\begin{array}{r} 39 \\ 4 \overline{)156} \\ -12 \\ \hline 36 \\ -36 \\ \hline 0 \end{array}$$

Check: Does $4 \cdot 39 = 156$?

$$\begin{array}{r} 39 \\ \times 4 \\ \hline 156 \end{array}$$ √

YTI #6 Solve each equation by using the Division Property of Equality. Use Examples 7 and 8 as guides. Show each step.

a) $n \cdot 5 = 30$ b) $21 = 3 \cdot n$ c) $12 \cdot n = 3{,}768$ d) $675 = 25 \cdot n$

In some equations, the variable is already isolated on one side, but the other side must be evaluated, as shown in Example 9.

Example 9:	Solve the equation $36 \cdot 5 = n$.	
Procedure:	Evaluate the left side. Here, the only check necessary is our multiplication work.	
Answer:	$36 \cdot 5 = n$	Because n is already isolated (on the right side) we simply evaluate the right side; apply multiplication directly. $\begin{array}{r} 36 \\ \times\ 5 \\ \hline 180 \end{array}$
	$180 = n$	
	Write n on the left: $n = 180$	So, the solution is 180.

YTI #7 Solve the equation. Use Example 9 as a guide.

a) $n = 25 \cdot 32$

b) $40 \cdot 15 = n$

You Try It Answers

YTI #1:	a)	18 = 16 No.	b)	20 = 20 Yes.	c)	50 = 50 Yes.	d)	40 = 45 No.
YTI #2:	a)	n = 4	b)	n = 13	c)	n = 28	d)	n = 12
YTI #3:	a)	n = 6	b)	n = 39				
YTI #4:	a)	n = 144	b)	n = 1,026				
YTI #5:	a)	5	b)	7	c)	3	d)	9
YTI #6:	a)	n = 6	b)	n = 7	c)	n = 314	d)	n = 27
YTI #7:	a)	n = 800	b)	n = 600				

Focus Exercises

For each, replace n with 16 and decide whether 16 is the solution.

1. $29 + n = 55$ **2.** $114 = n + 98$ **3.** $5 \cdot n = 80$ **4.** $210 = n \cdot 15$

Solve each equation. Check each answer to show that it is the solution.

5. $8 + n = 21$

6. $16 + n = 49$

7. $63 = n + 28$

8. $129 = n + 45$

9. $n = 38 + 84 + 76$

10. $925 + 110 + 640 = n$

11. $191 + 186 + n = 500$

12. $1{,}000 = 306 + 471 + n$

13. $2{,}817 + 3{,}199 = n$

14. $4{,}608 + 3{,}392 = n$

15. $5{,}208 + 3{,}691 + n = 10{,}000$

16. $8{,}156 + 7{,}519 + n = 20{,}000$

17. $n \cdot 3 = 57$

18. $52 = n \cdot 4$

19. $98 = 7 \cdot n$

20. $8 \cdot n = 96$

21. $n \cdot 12 = 156$

22. $405 = 15 \cdot n$

23. $37 \cdot 40 = n$

24. $n = 23 \cdot 30$

25. $25 \cdot n = 800$

26. $700 = 35 \cdot n$

27. $n \cdot 53 = 2{,}491$

28. $n \cdot 48 = 3{,}168$