Section 5.5 Dividing Decimals

Objectives
In this section, you will learn to:

- Divide decimal numbers by whole numbers.
- Divide decimal numbers by decimals.
- Write fractions as decimal numbers.
- Divide decimal numbers by powers of 10.

To successfully complete this section,
you need to understand:

- Dividing whole numbers (1.4)
- Repeating decimals (5.1)
- Terminating decimals (5.1)
- Writing decimal fractions as decimals (5.1)
- Multiplying decimals by powers of 10 (5.4)

INTRODUCTION

We know that 40 ÷ 4 = 10. What about 36 ÷ 4?

If we think about it, 36 is a little bit less than 40, so the quotient of 36 ÷ 4 should be a little bit less than 10, and it is: 36 ÷ 4 = 9.

Likewise, the quotient of 52 ÷ 4 should be a little bit *more* than 10 because 52 is a little bit more than 40. In fact, 52 ÷ 4 = 13: →

$$\begin{array}{r} 13 \\ 4\,\overline{)\,52} \\ \underline{-\,4} \\ 12 \\ \underline{-\,12} \\ 0 \end{array}$$

We can check this by multiplying:

$$\begin{array}{r} 13 \\ \underline{\text{x}\ \ 4} \\ 52 \end{array}$$

We also know that 4 ÷ 4 = 1. What could we say about 5.2 ÷ 4? Using the same reasoning, because 5.2 is a little bit more than 4, the quotient of 5.2 ÷ 4 should be a little bit more than 1, and it is:
5.2 ÷ 4 = 1.3.

The point is this: when we divide a decimal number by a whole number, the quotient will also be a decimal number.

$$\begin{array}{r} 1.3 \\ 4\,\overline{)\,5.2} \\ \underline{-\,4} \\ 1\ 2 \\ \underline{-\,1\ 2} \\ 0 \end{array}$$

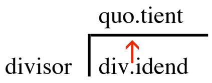
We can check this by multiplying:

$$\begin{array}{r} 1.3 \\ \underline{\text{x}\ \ 4} \\ 5.2 \end{array}$$

DIVIDING DECIMAL NUMBERS BY WHOLE NUMBERS

Dividing Decimal Numbers by Whole Numbers Using Long Division

1. Place a decimal point in the quotient directly above the decimal point in the dividend.
2. Divide as if each were a whole number.
3. Place zeros at the end of the dividend if needed.



Example 1: Divide using long division: 6.52 ÷ 4

Procedure: When the divisor (4) is a whole number and the dividend (6.52) has a decimal point in it:

(1) we place a decimal point in the quotient directly above the decimal point in the dividend; and

(2) we ignore the decimal point and divide as if each were a whole number.

Answer:

$$\begin{array}{r} 1.63 \\ 4\overline{)\,6.52} \\ \underline{-4} \\ 25 \\ \underline{-24} \\ 12 \\ \underline{-12} \\ 0 \end{array}$$

Notice that the decimal point in the quotient is directly above the decimal point in the dividend.

6.52 ÷ 4 = 1.63

Check by multiplying:

$$\begin{array}{r} 1.63 \\ \underline{\times\ 4} \\ 6.52 \end{array}$$

YTI #1 Divide using long division. Use Example 1 as a guide.

a) 6.92 ÷ 4

b) 10.482 ÷ 6

c) 50.4 ÷ 8

Recall from Section 1.4 that if the divisor doesn't divide into the first digit, we can place a 0 above the first digit and then continue to divide into the first two digits, and so on.

Example 2: Divide using long division.

a) $0.741 \div 3$

b) $\dfrac{0.354}{6}$

Procedure:

a)
$$\begin{array}{r} 0.247 \\ 3\,\overline{)\,0.741} \\ \underline{-\,6} \\ 14 \\ \underline{-\,12} \\ 21 \\ \underline{-\,21} \\ 0 \end{array}$$

b)
$$\begin{array}{r} 0.059 \\ 6\,\overline{)\,0.354} \\ \underline{-\,0} \\ 35 \\ \underline{-\,30} \\ 54 \\ \underline{-\,54} \\ 0 \end{array}$$

(b) We place the decimal point in the quotient, along with a 0 in the whole number place.

Because 6 won’t divide into 3, this quotient requires a 0 above the 3 as well.

Answer: $0.741 \div 3 = 0.247$ $\qquad \dfrac{0.354}{6} = 0.059$

Notice that the answer for a) has a 0 *before* the decimal point—in the whole number position, but the answer for b) has one 0 *before* and one 0 *immediately after* the decimal point.

Caution: It is common that the quotient is written with a whole number, even if it is 0. The whole number will be 0 whenever the dividend is less than the divisor.

YTI #2 Divide using long division. Use Example 2 as a guide.

a) $1.099 \div 7$

b) $\dfrac{0.056}{8}$

c) $1.1415 \div 15$

Sometimes there are not enough decimal places in the dividend to divide fully. When that happens, you need only place as many zeros as you wish at the end of the dividend. This way, you can continue to divide as long as you need to.

Example 3: Divide using long division: 1.37 ÷ 2

Procedure: Extend the number of decimal places in the dividend by placing as many zeros at the end of the decimal as needed.

Answer:

$$\begin{array}{r} 0.685 \\ 2\,\overline{)\,1.3700} \\ \underline{-\,1\,2} \\ 17 \\ \underline{-\,16} \\ 10 \\ \underline{-\,10} \\ 0 \end{array}$$

We need to place only one extra zero at the end of the dividend.

However, it's okay to place more (two extra zeros are shown here). Usually, we don't know ahead of time how many zeros we will actually need. Sometimes, we may need to add more than we originally thought.

Once we get a remainder of zero, we're finished.

1.37 ÷ 2 = 0.685

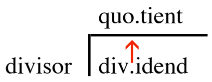
YTI #3

Divide using long division. Use Example 3 as a guide. (In each of these, you'll need to place at least one zero at the end of the dividend.)

a) 3.7 ÷ 5

b) $\frac{0.5}{4}$

c) 0.77 ÷ 8



Think about it #1: When dividing into a decimal, when is it necessary to add zeros to the end of the dividend?

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Sometimes we can place as many zeros as we wish, but we never get a remainder of 0. When this happens, the quotient is a repeating decimal, as you will see in the next example. The repeating pattern will begin to become clear when you get a recurring remainder. Once you see the pattern, you can stop dividing and place a bar over the repeating digits (the repetend), just as you did in Section 5.1.

Example 4: Divide using long division: 2.5 ÷ 3

Procedure: You can probably see that—temporarily ignoring the decimal—3 will not divide evenly into 25. We will need to extend the number of decimal places in the dividend by placing some zeros at the end.

This time, however, we're going to get a repeating decimal. When you recognize that the pattern is repeating, write the quotient with the bar over the repeating part.

Answer:

$$\begin{array}{rl}
0.833 & \\
3\,\overline{)\,2.5000} & \leftarrow \text{Place plenty of zeros at the end of 2.5.} \\
\underline{-\,2\,4} & \\
10 & \leftarrow \text{The remainder is 1; bring down the 0.} \\
\underline{-\,9} & \\
10 & \leftarrow \text{The remainder is 1; bring down the 0.} \\
\underline{-\,9} & \\
1 & \leftarrow \text{The remainder of 1 is recurring, let's stop.}
\end{array}$$

$2.5 \div 3 = 0.8\overline{3}$

Sometimes the pattern takes a little longer to develop.

Example 5: Divide using long division: $1.62 \div 11$

Procedure: This time, the recurring decimal will take two decimal places. Watch!

Answer:

$$
\begin{array}{rl}
0.147272 & \\
11\,\overline{)\,1.62000} & \leftarrow \text{Place plenty of zeros at the end of 1.62.} \\
\underline{-\,1\,1} & \\
52 & \leftarrow \text{The remainder is 5; bring down the 2.} \\
\underline{-\,44} & \\
80 & \leftarrow \text{The remainder is 8; bring down the 0.} \\
\underline{-\,77} & \\
30 & \leftarrow \text{The remainder is 3; bring down the 0.} \\
\underline{-\;\;22} & \\
80 & \leftarrow \text{This remainder looks familiar; bring down the 0.} \\
\underline{-\,77} & \\
30 & \leftarrow \text{So does this one; let's stop.}
\end{array}
$$

The pattern of 72 repeats in the quotient. $1.62 \div 11 = 0.14\overline{72}$

YTI #4 Divide using long division. Use Examples 4 and 5 as guides.

a) $6.1 \div 9$

b) $0.43 \div 6$

c) $\frac{15.54}{33}$

DIVIDING WHEN THE DIVISOR CONTAINS A DECIMAL

To this point, you have learned to divide a decimal number by a whole number, such as $6.52 \div 4$ (as was demonstrated in Example 1). What if we need to divide 6.52 by the decimal number 0.4 instead of the whole number 4?

$$\text{divisor}\,\overline{\big)\,\text{dividend}}^{\;\text{quotient}}$$

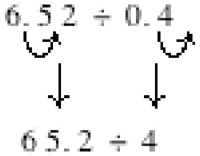
dividend ÷ divisor = quotient

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$$

To answer this question, we will use these three ideas to help us develop a procedure for dividing by a decimal number:

(1) Any division can be written as a fraction: $6.52 \div 0.4 = \dfrac{6.52}{0.4}$

(2) We can multiply any decimal number by a power of 10, moving the decimal point to the right: $\begin{cases} 6.52 \times 10 = 65.2 \\ \text{and } 0.4 \times 10 = 4 \end{cases}$

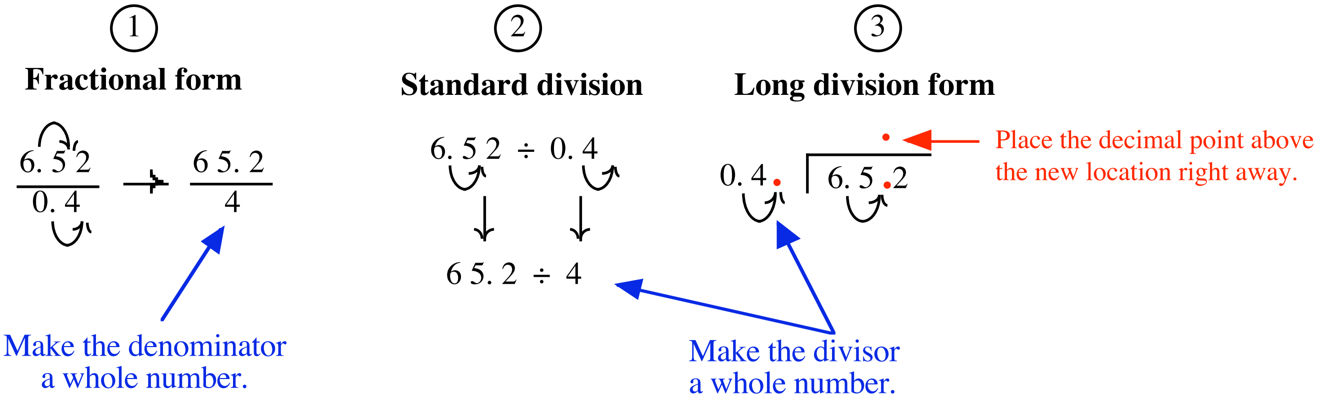


(3) We can multiply any fraction by a form of 1 without changing the value: $\dfrac{6.52}{0.4} \times \dfrac{10}{10} = \dfrac{65.2}{4}$, which is $65.2 \div 4$.

Notice that we have changed $6.52 \div 0.4$ into $65.2 \div 4$ by adjusting the decimal point in *each* number, moving it one place to the right: →

The point is this: If the divisor is a decimal number, then we can make it a whole number by moving the decimal point in both the dividend *and* the divisor by the same number of places.

In 6.52 ÷ 0.4, we can move each decimal point one place to the right in one of three settings:

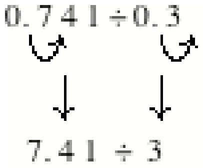


Whichever setting you choose, the actual division should be done using long division.

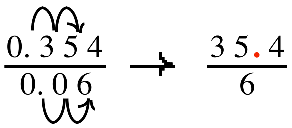
Example 6: Divide. a) 0.741 ÷ 0.3 b) $\frac{0.354}{0.06}$

Procedure: First adjust each number so that the divisor is a whole number:

a) Because the divisor has one decimal place, move each decimal point one place to the right:



b) Because the divisor has two decimal places, move each decimal point two places to the right:



Now set up the long division using these adjusted numbers.

Answer: a)

$$\begin{array}{r} 2.47 \\ 3\overline{)\,7.41} \\ \underline{-\ 6} \\ 14 \\ \underline{-12} \\ 21 \\ \underline{-21} \\ 0 \end{array}$$

0.741 ÷ 0.3 = 2.47

b)

$$\begin{array}{r} 5.9 \\ 6\overline{)\,35.4} \\ \underline{-\ 30} \\ 5\,4 \\ \underline{-5\,4} \\ 0 \end{array}$$

0.354 ÷ 0.06 = 5.9

YTI #5 Divide. Adjust the decimal points appropriately to make the divisor into a whole number. Use Example 6 as a guide.

a) $0.63 \div 0.9$

b) $\dfrac{4.85}{0.05}$

c) $0.748 \div 0.0011$

DIVIDING DECIMAL NUMBERS BY POWERS OF 10

As we saw in Section 5.4, multiplying a decimal by a power of 10 has the effect of moving the decimal point to the right. As we'll see next, *dividing* by a power of 10 has the opposite effect and moves the decimal point to the left.

To understand why we would move the decimal point to the left, consider 54.7 ÷ 100. We can first write this as a fraction, $\dfrac{54.7}{100}$, and then multiply this by a form of 1, $\dfrac{10}{10}$, so that the numerator becomes a whole number:

$$\frac{54.7}{100} \cdot \frac{10}{10} = \frac{54.7 \text{ x } 10}{100 \text{ x } 10} = \frac{547}{1000} = 0.547$$

Notice that multiplying the fraction by $\dfrac{10}{10}$ has two effects, (1) it makes the numerator a whole number by moving the decimal point one place to the right, and (2) it increases the denominator to a higher power of 10.

Here are two examples showing the moving of the decimal point directly:

a) $\frac{3.65}{10} = 0.365$

$\underset{\curvearrowleft}{.\,3}\,.\,6\,5 = 0.365$

Dividing by 10, which has only one zero, has the effect of moving the decimal point of 3.65 one place to the left. We can write in 0 as the whole number.

b) $45.2 \div 1{,}000 = 0.0452$

$\downarrow$

$00045.2 \div 1{,}000 = 0.0452$

$0\,0.\underset{\curvearrowleft\curvearrowleft\curvearrowleft}{0\,4\,5}\,.\,2 = 0.0452$

Dividing by 1,000 suggests that we need to move the decimal point in 45.2 three places to the left.

However, there aren't enough whole numbers in 45.2, so we must place some zeros at the beginning of it so that there is someplace to move the decimal point to.

Here is the procedure that results from all of this:

Dividing a Decimal by a Power of Ten

(1) Count the number of zeros in the power of ten.

(2) Move the decimal point that many places to the left.

YTI #6 Divide by moving the decimal point the appropriate number of places to the left.

a) $8.7 \div 10$

b) $\frac{34.6}{10}$

c) $\frac{76.4}{100}$

d) $20.9 \div 1{,}000$

WRITING FRACTIONS AS DECIMAL NUMBERS

Any fraction in which both the numerator and denominator are whole numbers can be written as a decimal number, either a terminating decimal or a repeating decimal.

Start by writing the fraction as division, then divide using long division. For example, $\frac{3}{4} = 3 \div 4$.

Because 4 won't divide into 3 directly, it is necessary to write 3 as 3.000 (possibly with more or with fewer ending zeros) in order to divide.

Example 7: Find the decimal equivalent of each fraction. a) $\frac{3}{4}$ b) $\frac{5}{6}$

Procedure: Write the fraction as division and then divide using long division. Write each numerator with a decimal point followed by some ending zeros.

Answer: a) $\frac{3}{4}$ can be thought of as $3 \div 4$ b) $\frac{5}{6}$ can be thought of as $5 \div 6$

$$\begin{array}{r} 0.75 \\ 4\overline{)3.000} \\ \underline{-2\,8} \\ 20 \\ \underline{-20} \\ 0 \end{array} \qquad\qquad \begin{array}{r} 0.8333 \\ 6\overline{)\,5.0000} \\ \underline{-4\,8} \\ 20 \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 20 \end{array}$$

$\frac{3}{4} = 0.75$, a terminating decimal $\frac{5}{6} = 0.8\overline{3}$, a repeating decimal

YTI #7 Find the decimal equivalent of each fraction. Use Example 7 as a guide.

a) $\frac{6}{5}$ b) $\frac{4}{9}$ c) $\frac{23}{40}$

A STRATEGY FOR DIVIDING WITH DECIMAL NUMBERS

Here is the strategy for dividing with decimals:

(1) Divide only by a whole number; if the divisor is a decimal number, then adjust both the dividend and divisor by moving each decimal point an appropriate number of places to the right.

(2) If the quotient has no whole number, write 0 in the quotient.

(3) Add as many zeros as needed to the end of any decimal dividend to continue to divide. Sometimes the quotient will be a terminating decimal, and other times it will be a repeating decimal.

Example 8: Follow the strategy guidelines, above, to divide.

a) $0.375 \div 0.4$

b) $29 \div 0.03$

Procedure: (1) Adjust the dividend and divisor:

a) Move each decimal point one place to the right.

$0.\underset{\curvearrowright}{3}.75 \div 0.\underset{\curvearrowright}{4}. \rightarrow 3.75 \div 4$

b) Move each decimal point two places to the right (give 29 a decimal point and some zeros).

$29.\underset{\curvearrowright\curvearrowright}{00}. \div 0.\underset{\curvearrowright\curvearrowright}{03}. \rightarrow 2{,}900 \div 3$

(2) Only the quotient in part (a) requires a zero as the whole number.

(3) Add zeros to the end of each dividend for further division.

Answer:

a)

$$\begin{array}{r} 0.9375 \\ 4\,\overline{)\,3.7500} \\ \underline{-3\,6} \\ 15 \\ \underline{-12} \\ 30 \\ \underline{-28} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

b)

$$\begin{array}{r} 0966.66 \\ 3\,\overline{)\,2900.00} \\ \underline{-27} \\ 20 \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 20 \end{array}$$

(repeats....)

(b) Notice that the recurring remainder, 2, began right away, but we can't abbreviate this repeating digit (with a bar over the 6) until *after* the decimal point in the quotient.

$0.375 \div 0.4 = 0.9375$

$29 \div 0.03 = 966.\overline{6}$

YTI #8 Divide. Use Example 8 as a guide.

a) $0.523 \div 8$

b) $\frac{8.32}{0.3}$

c) $0.079 \div 0.05$

d) $7 \div 0.004$

e) $4.1062 \div 0.009$

f) $\frac{3.75}{1.1}$

Below is a chart of the equivalent decimal values of some common fractions.

Decimal Equivalents of Some Fractions			
$\frac{1}{2} = 0.5$	$\frac{1}{5} = 0.2$	$\frac{5}{6} = 0.8333...$	$\frac{1}{9} = 0.111...$
$\frac{1}{3} = 0.333...$	$\frac{2}{5} = 0.4$	$\frac{1}{8} = 0.125$	$\frac{2}{9} = 0.222...$
$\frac{2}{3} = 0.666...$	$\frac{3}{5} = 0.6$	$\frac{3}{8} = 0.375$	$\frac{4}{9} = 0.444...$
$\frac{1}{4} = 0.25$	$\frac{4}{5} = 0.8$	$\frac{5}{8} = 0.625$	$\frac{5}{9} = 0.555...$
$\frac{3}{4} = 0.75$	$\frac{1}{6} = 0.1666...$	$\frac{7}{8} = 0.875$	$\frac{7}{9} = 0.777...$

Think about it #2: Based on the chart above,

a) What is the decimal equivalent of $\frac{8}{9}$? __________________

b) What is the decimal equivalent of $\frac{3}{9}$? __________________

c) Does the decimal equivalent of $\frac{3}{9}$ appear somewhere else in the chart? Explain your answer.

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You Try It Answers: Section 5.5

YTI #1:	a) 1.73	b) 1.747	c) 6.3	
YTI #2:	a) 0.157	b) 0.007	c) 0.0761	
YTI #3:	a) 0.74	b) 0.125	c) 0.09625	
YTI #4:	a) $0.6\overline{7}$	b) $0.071\overline{6}$	c) $0.47\overline{09}$	
YTI #5:	a) 0.7	b) 97	c) 680	
YTI #6:	a) 0.87	b) 3.46	c) 0.764	d) 0.0209
YTI #7:	a) 1.2	b) $0.\overline{4}$	c) 0.575	
YTI #8:	a) 0.065375	b) $27.7\overline{3}$	c) 1.58	
	d) 1,750	e) $456.2\overline{4}$	f) $3.4\overline{09}$	

Focus Exercises: Section 5.5 w/Answers

Think Again.

1. When dividing into a decimal, when is it necessary to add zeros to the end of the dividend? *(Think about it #1)*

When the dividing still has a remainder (other than 0), and the dividend has no more digits, then it is appropriate to add 0's at the end of the dividend.

2. Can a repeating decimal be a divisor? Explain your answer or show an example that supports your answer.

No. (Not in its decimal form.) We cannot move the decimal point to the end of the repeating decimal to start the division process.

3. Can a repeating decimal be a dividend? Explain your answer or show an example that supports your answer.

Yes, as long as the divisor is a terminating decimal. The quotient will be a repeating decimal.

4. In the chart at the end of the section, we see that when 9 is a denominator, we get repeating decimal values, such as $\frac{1}{9} = 0.11111...$ and $\frac{2}{9} = 0.2222...$ and so on. Using this reference, what is the decimal equivalent of $\frac{13}{9}$?

1.4444... The fraction must first be written as a mixed number.

Divide.

5. $5.4 \div 6$
0.9

6. $3.8 \div 2$
1.9

7. $\frac{61.5}{5}$
12.3

8. $74.1 \div 3$
24.7

9. $\frac{87.03}{9}$
9.67

10. $51.96 \div 4$
12.99

11. $\frac{0.95}{5}$
0.19

12. $0.63 \div 7$
0.09

13. $1.312 \div 8$
0.164

14. $1.737 \div 9$
0.193

15. $2.67 \div 5$
0.534

16. $\frac{3.09}{5}$
0.618

17. $1.62 \div 4$
0.405

18. $2.38 \div 4$
0.595

19. $0.532 \div 8$
0.0665

20. $0.676 \div 8$
0.0845

21. $\frac{4.7}{3}$
$1.5\overline{6}$

22. $\frac{6.5}{3}$
$2.1\overline{6}$

23. $4.8 \div 9$
$0.5\overline{3}$

24. $5.7 \div 9$
$0.6\overline{3}$

25. $3.4 \div 11$
$0.3\overline{09}$

26. $\frac{6.2}{11}$
$0.5\overline{63}$

27. $0.49 \div 15$
$0.032\overline{6}$

28. $0.35 \div 6$
$0.058\overline{3}$

Divide.

29. $\frac{2.16}{0.6}$
3.6

30. $3.42 \div 0.9$
3.8

31. $0.468 \div 0.4$
1.17

32. $0.736 \div 0.8$
0.92

33. $0.198 \div 0.05$
3.96

34. $0.267 \div 0.05$
5.34

35. $5.6 \div 0.08$
70

36. $3.6 \div 0.09$
40

37. $\frac{48}{0.06}$
800

38. $125 \div 0.05$
2,500

39. $0.24 \div 0.012$
20

40. $0.96 \div 0.032$
30

41. $0.126 \div 0.3$
0.42

42. $\frac{0.535}{0.5}$
1.07

43. $0.07128 \div 0.4$
0.1782

44. $0.0579 \div 0.3$
0.193

45. $0.0455 \div 0.09$
$0.50\overline{5}$

46. $0.0316 \div 0.03$
$1.05\overline{3}$

47. $\frac{0.073}{0.11}$
$0.6\overline{63}$

48. $0.038 \div 0.11$
$0.3\overline{45}$

49. $0.0015 \div 0.008$
0.1875

50. $0.0053 \div 0.004$
1.325

51. $0.062 \div 0.25$
0.248

52. $\frac{0.021}{0.75}$
0.028

Divide by moving the decimal point the appropriate number of places to the left.

53. $25.8 \div 10$
2.58

54. $90.3 \div 10$
9.03

55. $\frac{45.7}{10}$
4.57

56. $\frac{3.9}{10}$
0.39

57. $74.9 \div 100$
0.749

58. $2.36 \div 100$
0.0236

59. $\frac{6.4}{100}$
0.064

60. $\frac{0.3}{100}$
0.003

61. $0.45 \div 100$
0.0045

62. $1.03 \div 100$
0.0103

63. $42.1 \div 1{,}000$
0.0421

64. $8.2 \div 1{,}000$
0.0082

Find the decimal equivalent of each fraction.

65. $\frac{5}{2}$
2.5

66. $\frac{9}{2}$
4.5

67. $\frac{7}{4}$
1.75

68. $\frac{13}{4}$
3.25

69. $\frac{8}{5}$
1.6

70. $\frac{11}{5}$
2.2

71. $\frac{9}{8}$
1.125

72. $\frac{11}{8}$
1.375

73. $\frac{13}{20}$
0.65

74. $\frac{29}{20}$
1.45

75. $\frac{17}{25}$
0.68

76. $\frac{43}{25}$
1.72

77. $\frac{7}{6}$
$\mathbf{1.1\overline{6}}$

78. $\frac{11}{6}$
$\mathbf{1.8\overline{3}}$

79. $\frac{11}{9}$
$\mathbf{1.\overline{2}}$

80. $\frac{23}{9}$
$\mathbf{2.\overline{5}}$

81. $\frac{9}{11}$
$\mathbf{0.\overline{81}}$

82. $\frac{4}{11}$
$\mathbf{0.\overline{36}}$

83. $\frac{8}{15}$
$\mathbf{0.5\overline{3}}$

84. $\frac{19}{15}$
$\mathbf{1.2\overline{6}}$

85. $\frac{4}{37}$
$\mathbf{0.\overline{108}}$

86. $\frac{5}{37}$
$\mathbf{0.\overline{135}}$

87. $\frac{3}{7}$
$\mathbf{0.\overline{428571}}$

86. $\frac{1}{7}$
$\mathbf{0.\overline{142857}}$

Use your knowledge of dividing signed numbers, along with the techniques of dividing decimals, to find each quotient.

89. $-0.24 \div 8$

-0.03

90. $-0.36 \div 4$

-0.09

91. $\frac{-17}{4}$

-4.25

92. $\frac{-27}{5}$

-5.4

93. $2.88 \div (-0.03)$

-96

94. $0.198 \div (-0.06)$

-3.3

95. $\frac{16}{-9}$

$\mathbf{-1.\overline{7}}$

96. $\frac{38}{-11}$

$\mathbf{-3.\overline{45}}$

97. $-3.62 \div (-0.05)$

72.4

98. $-15.3 \div (-0.04)$

382.5

99. $\frac{-41}{-6}$

$\mathbf{6.8\overline{3}}$

100. $\frac{-31}{-9}$

$\mathbf{3.\overline{4}}$

Think Outside the Box.

101. Each of these fractions can be written as a terminating decimal, as shown.

$$\frac{3}{4} = 0.75,\ \frac{1}{8} = 0.125,\ \frac{9}{10} = 0.9,\ \frac{7}{16} = 0.4275,\ \frac{12}{25} = 0.48,$$

$$\frac{13}{40} = 0.325,\ \frac{27}{50} = 0.54, \frac{87}{100} = 0.87, \text{ and } \frac{94}{125} = 0.752.$$

Find the prime factorization of the denominator of each of these fractions. What does each prime factorization have in common?

$$\mathbf{\frac{3}{2 \cdot 2}, \frac{1}{2 \cdot 2 \cdot 2}, \frac{9}{2 \cdot 5}, \frac{7}{2 \cdot 2 \cdot 2 \cdot 2}, \frac{12}{5 \cdot 5}, \frac{13}{2 \cdot 2 \cdot 2 \cdot 5}, \frac{27}{2 \cdot 5 \cdot 5}, \frac{87}{2 \cdot 2 \cdot 5 \cdot 5}, \frac{94}{5 \cdot 5 \cdot 5}}$$

Each denominator has only 2 and 5 as prime factors.

102. Each of these fractions can be written as a repeating decimal, as shown.

$$\frac{2}{9} = 0.\overline{2},\ \frac{3}{14} = 0.2\overline{142857},\ \frac{13}{22} = 0.5\overline{90},\ \frac{7}{12} = 0.58\overline{3},$$

$$\frac{10}{33} = 0.\overline{30},\ \frac{24}{55} = 0.4\overline{36},\ \frac{49}{66} = 0.7\overline{42}, \text{ and } \frac{64}{75} = 0.85\overline{3}.$$

Find the prime factorization of the denominator of each of these fractions. What does each prime factorization have in common?

$$\mathbf{\frac{2}{3 \cdot 3}, \frac{3}{2 \cdot 7}, \frac{13}{2 \cdot 11}, \frac{7}{2 \cdot 2 \cdot 3}, \frac{24}{5 \cdot 7}, \frac{38}{5 \cdot 11}, \frac{49}{2 \cdot 3 \cdot 11}, \frac{61}{2 \cdot 2 \cdot 3 \cdot 7}, \frac{64}{3 \cdot 5 \cdot 5}}$$

Each denominator has prime factors other than 2 and 5.

103. Based on your answers to #97 and #98, how can you determine if a fraction will be a terminating decimal or a repeating decimal?

If a fraction's denominator has only 2 or 5 as prime factors, then the fraction can be written as a terminating decimal. If a fraction's denominator has a prime factor other than 2 or 5, then the fraction can be written as a repeating decimal.