Solving Linear Equations (in one variable)

In Chapter 2 of my Elementary Algebra text you are introduced to solving linear equations. The main idea presented throughout Sections 2.1 – 2.3 is that you need to ***isolate the variable***.

In Section 2.1 you are shown how to solve equations with ***just one operation***. These are the simplest types of equations to solve:

(A) $3x = -15$ $\quad -4 = x - 6$ $\quad \frac{2}{3}x = 12$ $\quad$ and so on.

Solve these Type A equations

In Section 2.2 you are first taught to solve equations with ***two operations***, such as

(B) $3x + 7 = 25$ $\quad 4 - 2x = -6$ $\quad \frac{3}{4}x - 5 = 19$

solve these Type B equations

The first step in solving these equations is to get them to look like the equations in (A) – those with only one operation.

Later on in Section 2.2 you learn how to solve more complicated equations, ones that have ***more than one variable term***, such as

(C) $-3x - 14 = x + 10$

solve this Type C equation

In these equations, our first step is to get the variable terms together and make them look like the equations in (B).

Do you get the pattern here? With every more complicated equation, we try to get it to be less complicated by manipulating it so that it looks like the type before it.

So, what happens when the equation has parentheses in it? For example, $2(x - 5) + 7 = -9 - (4x + 6)$

solve this Type C equation

Well, we multiply — distribute — to "clear" the parentheses and try to manipulate it to look like the equation in (C),

then manipulate it some more to make it look like an equation in (B),

and then some more to make it look like an equation in (A)

and then finally solve it by *isolating the variable*.

All along the way we develop a set of rules that work the equation toward isolating the variable. With each new twist to the problem solving process, we add a new rule.

Click here to go to the bottom to see Solving Linear Equations, The Ultimate Guidelines

Equations Involving Fractions or Decimals

The next twist comes into play when fractions and decimals are involved, such as

(D) $x + \frac{2}{3}x = 15$ $\qquad -4 + \frac{1}{3}x = \frac{5}{2}x + 9$ $\qquad 0.6x - 1.25 = 0.4x + 0.35$

solve type D Equations

In each of these, the new rule is to "clear the fractions" (or decimals) by multiplying each side by the least common denominator (LCD). Once the denominator is cleared, the equations look like the one in (C), which will then be simplified to like one in (B), and so on.

Examples: Solving Type (A) Equations

(A) $3x = -15$ $\qquad -4 = x - 6$ $\qquad \frac{2}{3}x = 12$

Divide each side by 3 | Add + 6 to each side | Multiply each side by the reciprocal.

$\frac{3x}{3} = \frac{-15}{3}$ $\qquad \underline{+6 = +6}$ $\qquad \frac{3}{2}\cdot\frac{2}{3}x = 12\cdot\frac{3}{2}$

$x = -5$ $\qquad 2 = x$ $\qquad x = 18$

(√) Check each answer to make sure it is a solution:

(√) $3x = -15$ $\qquad -4 = x - 6$ $\qquad \frac{2}{3}x = 12$

$3(-5) = -15$?? $\qquad -4 = 2 - 6$?? $\qquad \frac{2}{3}\cdot 18 = 12$??

$-15 = -15$ √ $\qquad -4 = -4$ √ $\qquad 12 = 12$ √

- 5 is the solution | 2 is the solution | 18 is the solution

back to top

Examples: Solving type (B) Equations

(B) $3x + 7 = 25$ $\quad$ $4 - 2x = -6$ $\quad$ $\frac{3}{4}x - 5 = 19$

Add – 7 to each side $\quad$ Add - 4 to each side $\quad$ Add + 5 to each side

$\underline{-7 = -7}$ $\quad$ $\underline{-4 \quad = -4}$ $\quad$ $\underline{+5 = +5}$

$3x = 18$ $\quad$ $-2x = -10$ $\quad$ $\frac{3}{4}x = 24$

Notice that, in each case, we got the constant terms together on the same side. In doing so, we changed the equation into one having only one operation (Type A).

Now that these equations have only one operation, you can solve from here—like those in (A).

$3x = 18$ $\quad$ $-2x = -10$ $\quad$ $\frac{3}{4}x = 24$

Divide each side by 3 $\quad$ Divide each side by - 2 $\quad$ Multiply each side by $\frac{4}{3}$

$3x = 18$ $\quad$ $-2x = -10$ $\quad$ $\frac{3}{4}x = 24$

$\frac{3x}{3} = \frac{18}{3}$ $\quad$ $\frac{-2x}{-2} = \frac{-10}{-2}$ $\quad$ $\frac{4}{3} \cdot \frac{3}{4}x = 24 \cdot \frac{4}{3}$

$x = 6$ $\quad$ $x = 5$ $\quad$ $x = 32$

You should check each answer by placing it in the original equation,

back to top

Solving Type C Equations

Examples: Solving type (C) Equations

(C) $-3x - 14 = x + 10$

First, get the variable terms together on the same side; we can do this by either adding + 3x to each side or by adding - x to each side. The choice is yours.

I'm going to choose to add + 3x to each side—that will create a + 4x on the right side:

$$-3x - 14 = x + 10$$

$$\underline{+3x \qquad = +3x}$$

$$-14 = 4x + 10$$

We now have an equation like those in (B) and can solve from here (by getting the constants together on the same side).

$-14 = 4x + 10$ Add - 10 to each side

$\underline{-10 = \qquad -10}$

$-24 = 4x$ Divide each side by 4

$$\frac{-24}{4} = \frac{4x}{4}$$

$$-6 = x$$

(√) and we should check this answer—by placing it into the original equation—to make sure it is the solution:

$-3x - 14 = x + 10$ Check $x = -6$

$$-3(-6) - 14 = (-6) + 10$$

$$+18 - 14 = +4$$

$+4 = +4$ (√) - 6 is the solution.

back to top

Examples: Solving type (D) Equations

In this next set of equations, we need to clear the fractions or the decimals. We do so by multiplying each side by the LCD. This means multiplying each and every term by that LCD.

As for decimals, their denominators are either 10, 100 or 1,000, so the LCD is the same as the term with the most decimal places. In that case, each term should (by placing zeros at the end) be built up to have the same number of decimal places.

(D) $$x + \frac{2}{3}x = 15$$

The only denominator we see is 3 (the other terms have a denominator of 1), so the LCD is 3. Multiply each and every term by 3 or by $\frac{3}{1}$.

$$3 \cdot x + \frac{3}{1} \cdot \frac{2}{3}x = 3 \cdot 15$$

$$3x + 2x = 45$$ ← Combine like terms on the right side.

$$5x = 45$$ ← Divide each side by 5.

$$x = 9$$ ← Be sure to check this answer into the original equation.

(D) $$-4 + \frac{1}{3}x = \frac{5}{2}x + 9$$ ← The LCD is 6, so multiply each and every term by 6 or $\frac{6}{1}$.

$$6 \cdot (-4) + \frac{6}{1} \cdot \frac{1}{3}x = \frac{6}{1} \cdot \frac{5}{2}x + 6 \cdot 9$$

$$-24 + 2x = 15x + 54$$ ← Now this is just like the equation in (C), so you can use that as a guide to solve it from here (first get the variable terms together).

(D) $$0.6x - 1.25 = 0.4x + 0.35$$ ← The most decimal places in any one term is two places. So, build up each term to have two decimal places. Then, to clear the decimals, multiply each and every term by 100.

$$0.60x - 1.25 = 0.40x + 0.35$$ ← All terms now have two decimal places. Multiply each by 100. Multiplying by 100 has the effect of moving the decimal point two places to the right.

$$60x - 125 = 40x + 35$$ ← In this case, that means making each number an integer. Now this is just like the equation in (C), so you can use that as a guide to solve it from here (first get the variable terms together).

back to top

Other Examples:

$2(x - 5) + 7 = -9 - (4x + 6)$	Start by clearing the parentheses on each side --- the negative distributes to each term in $-(4x + 6)$
$2x - 10 + 7 = -9 - 4x - 6$	Combine like terms on each side.
$2x - 3 = -15 - 4x$ $+4x \quad = \quad +4x$	This is now a Type C equation; get the variable terms on the same side by either adding - 2x to each side or by adding + 4x to each side. I choose to add + 4x.
$6x - 3 = -15$ $+3 = +3$	This is now a Type B equation. Add + 3 to each side.
$6x = -12$	This is now a Type A equation. Divide each side by 6.

$$\frac{6x}{6} = \frac{-12}{6}$$

$$x = -2$$

And you should check the answer by placing - 2 into the original equation. That part is left to you.

This next equation is from my Elementary Algebra book, page 2.3 - 8 #d:

$.6(10x - 3) = 1.5(x + 2) - .3$	First, clear the parentheses by distributing.
$6x - 1.8 = 1.5x + 3 - .3$ $6.0x - 1.8 = 1.5x + 3.0 - 0.3$	Next, recognize that the most decimal places in any term is *one*, so build up each term to have one decimal place.
	Now multiply each side—each and every term—by 10 to clear the decimals.
$60x - 18 = 15x + 30 - 3$	Combine like terms on the right side; the left side is has not like terms to combine.
$60x - 18 = 15x + 27$ $-15x \quad = \quad -15x$	This is now a Type C equation; get the variable terms together. Add - 15x to each side.
$45x - 18 = 27$ $+18 = +18$	This is now a Type B equation; isolate the variable by adding + 18 to each side.
$45x = 45$	This is now a Type A equation; divide each side by 45. (I trust that you can divide without me showing you.)

$$x = 1$$

back to top

The Guidelines

Solving an equation can require a lot of steps sometimes, and there are rules for each step. Though each equation is different, there are still certain patterns that they follow. Here are my ultimate guidelines for solving linear equations:

Solving Linear Equations: The Ultimate Guidelines

The Preparation:

1. Eliminate any parentheses by distributing. (Be careful to look for negative multipliers.)
2. Clear any fractions or decimals by multiplying each side by the equation's LCD.
3. Combine like terms on each individual side.

Isolating the Variable:

4. If there is more than one variable term, get them together on the same side by adding the opposite of the one of them to each side.
5. Clear the operations; start with the main operation. Usually this means
   - i) adding to get the constant terms together on the same side (the side opposite the variable term)
   - ii) dividing each side by the coefficient.

Checking your answer:

6. Place the answer into the original equation to see if it gives a true result. If not, then search your problem solving for the error. (If the answer is a fraction or decimal, then sometimes the check is harder than the problem solving itself, so be careful!)

If you see any errors in this document, please let me know via e-mail: bob.prior@rcc.edu

back to top